Course Outline:

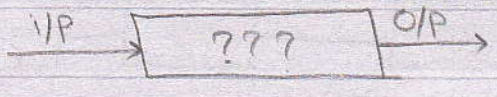
1. Introduction of FCS (examples)
2. Modeling Sys equations
   i) in the form of diff. eq's
   ii) in the form of state "
3. Solution of diff eq's
4. Frequency domain / Laplace
5. Block diagram representation / SFGs ✓
6. Stability & performance
7. Root locus design & analysis
8. Bode plot
9. PID Controllers.

3rd Jan 2014

Control System:

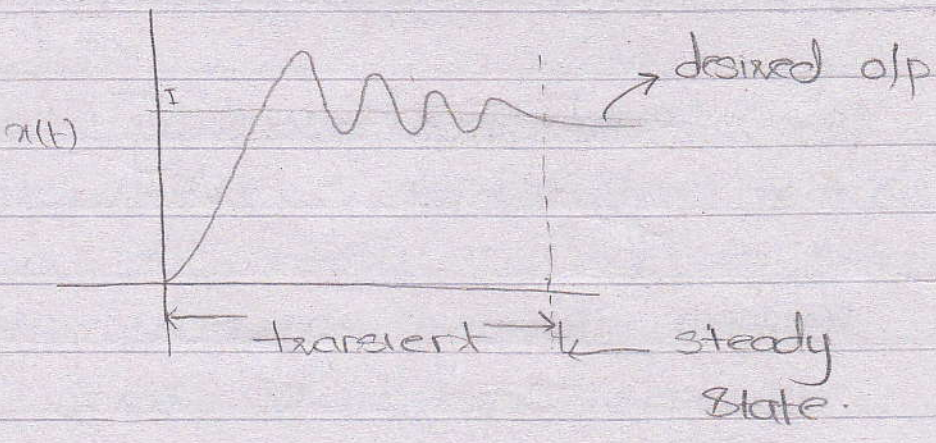
* Any device that allows certain i/p & gives same o/p is a system.

* To get a certain o/p we need a Control theory.



* Diff. eq have 2 kinds of soln.

  ↳ Complementary (Transient sol)
  ↳ Particular (steady-state sol)

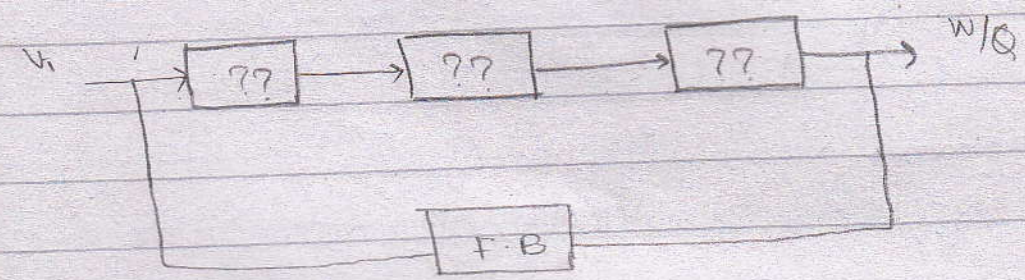


* When DE can't be solved easily, we'll use a mathematical tool called laplace.

* For certain specifications we need time domain analysis & for some we need freq domain analysis

FOR A DC MOTOR

Block diagram:



* There's a trade off b/w stability & performance in control sys.

* poles determine stability controlling than guarantees stability

* stability doesn't depend on zero's margins depend on zero's.

System

System is an awarangement set or collection of things connected or related in such a manner to form a complete object or act as a entire Unit.

Control:

The word Control is usually taken to mean to regulate, direct or Command.

Control System:

It is an arrangement of physical components connected, related in such a manner as to Command, direct or regulate itself or another system.

System Input:

It is the stimulus (excitation) applied to a Control system from an external source of energy in order to produce a specific response from the Control system.

Output:

The o/p is actual response obtained from a Control sys. It may or may not be

equal to the specified response implied by the i/p.

Types Of Control System:

1) Man Made System (switch, fans, etc)
2) Biological Or Any Other Natural System.
3) Hybrid Systems (Driving a car)

Classification Of Conrol System:

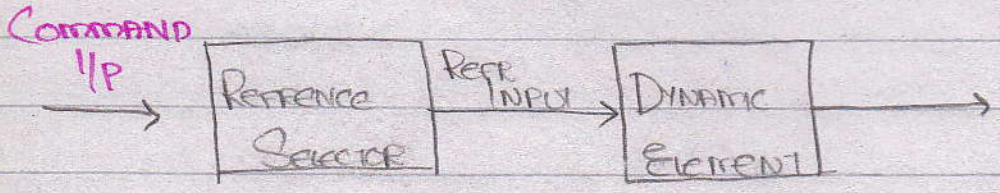
1.* Open Loop Control System (O.L.C.S)

i/p This Control action is independent of the output

OR

output has no effect on i/p

* Operator has to get involved in the sys.
For eg charging the temp of geezer acc to our requirement.



- Ref i/p is the exact i/p req for a certain o/p result.
- C.L.C.S can be converted into O.L.C.S

→ Features of OLCS

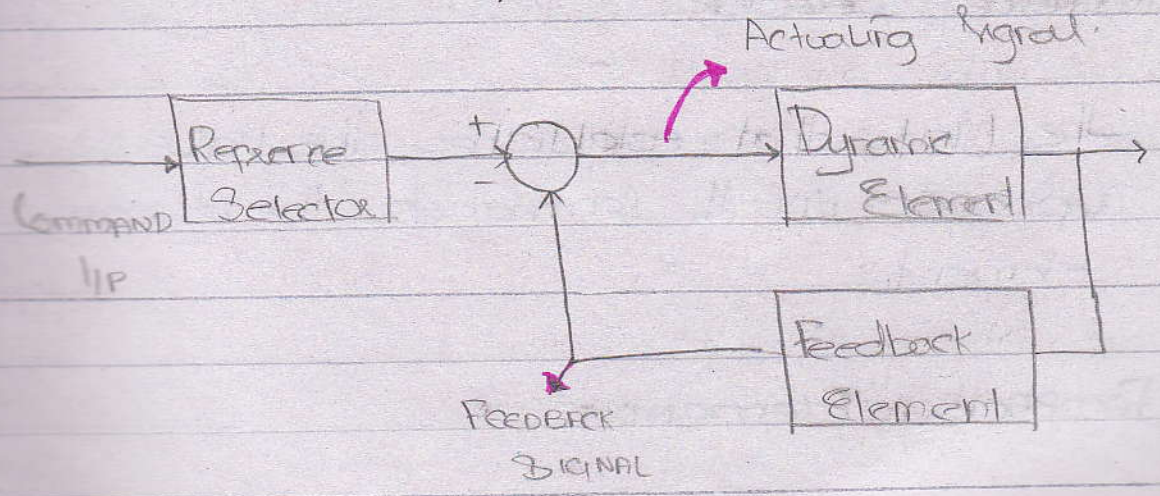
* Simple design
* less Expensive
* less actuate, accuracy depends upon Calibration.

2) Closed loop Control System (C.L.C.S)

Control action is independent on o/p in close loop control system.

OR

o/p has an effect on i/p.



→ FEATURES OF C.L.C.S:

* Complex design
* more expensive
* More Accurate
* Reduced Sensitivity b/w i/p & o/p
* Increased Bandwidth (freq at which system performs satisfactorily)

COMMAND INPUT:

The motivating i/p signals to the control system which is independent of the o/p.

REFRENCE SELECTOR:

The Unit that establishes the value of refrence i/p. It commands the dynamic element.

FEEDBACK ELEMENT:

A device that Compares i/p & o/p signal

Dynamic Element:

A Unit that reacts to an actuating signal to produce the desired o/p.

Actuating Signal:

The signal that is a difference b/w reference i/p & feedback signal.

Time Invariant & Time Variant System:

When the parameters of control system are satisfactory wrt time during the operation of a system. the sys is time invariant otherwise time variant systems

for eg fan is a time invariant systems whereas burning of coal & reusing it is time variant.

CHAPTER 2

WRITING EQUATIONS

1. MECHANICAL SYSTEMS:

They are called mechanical systems bec law of mechanics is involved. I/p & o/p both are mechanical

* CLASSIFICATION:

1) Mechanical translational sys
2) " rotational "

Elements of Interest	Energy Behavior	Force.
1) Mass (M)	$\frac{1}{2}MV^2$	$F = Ma$
2) Damper (D)	—	
3) Linear Spring (K)	$\frac{1}{2}Kx^2$	$F = BV$

* Damper works similar to a resistor.
$F = Kx$.

* These sys are those in wch motion takes place along a straight line.

The elements of interest are mass, linear Spring & damper. The variable of interest are displacement, velocity & acceleration.

PHYSICAL LAW:-

The sum of all forces acting on a point/node is equal to zero.

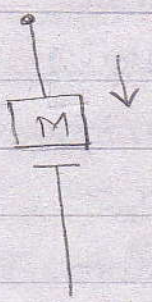
OR

The sum of all applied forces is equal to the sum of all reactive forces.

ELEMENTS OF INTEREST:

* MASS:

It is the element that stores the kinetic energy of translatory motion of the system.



$F = Ma$

$F = MDv$

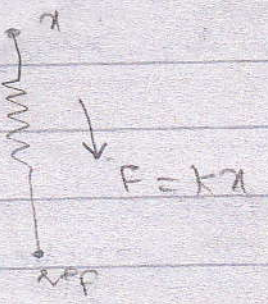
$F = MD^2x$

* One end of mass is always connected to reference b/c whenever it is displaced

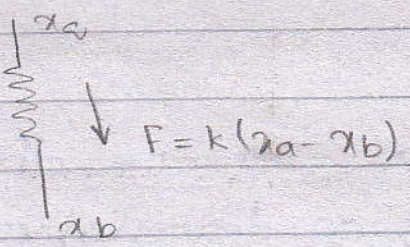
it moves completely.

* LINEAR SPRING:

It can be as actual spring, cable or a rod. It characterizes the element that stores the potential energy.



If in the case of no fix end



Reference point:

All systems are initially at rest. The end of spring & mass have position denoted as reference position, any displacement from this ref position is denoted as x, x_a, x_b etc.

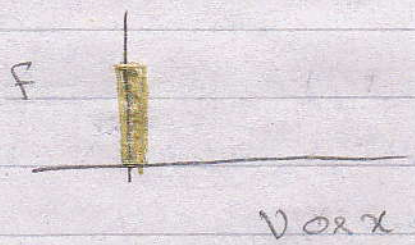
* DAMPER

FRICTION IN TRANSLATORY MOTION:

Whenever 2 bodies or objects passes over one another then the force of friction is blw the 2 contacting surfaces depending upon the composition of surfaces, pressure blw surfaces, & their relative velocities.

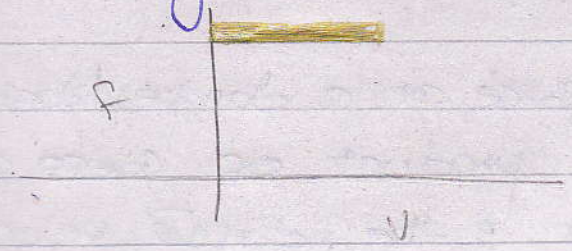
→ STATIC FRICTION:

It represents a retarding force that tends to present motion from beginning



→ COULOMB'S FRICTION:

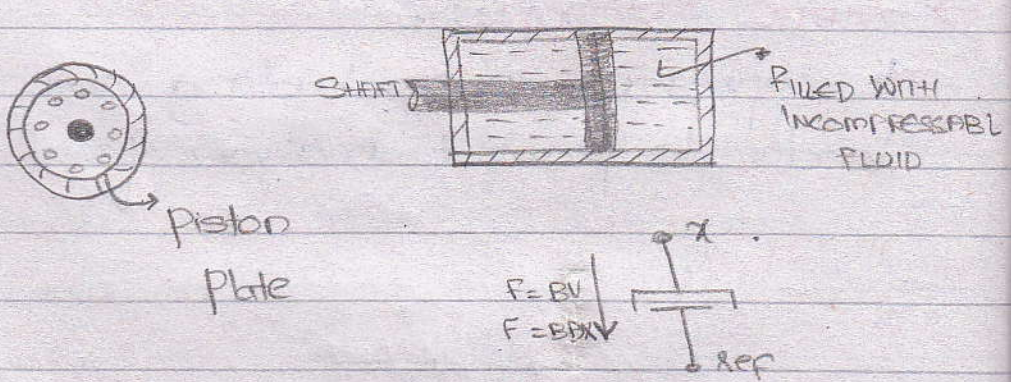
A retarding force that has a const magnitude wrt velocity.



-Viscous Friction

A retarding force that has linear relationship b/w the applied force & velocity. We can apply viscous friction with the help of damper.

Damping:



The decay of response wrt time is known as damping. A device that is used to apply damping (viscous friction) is known as damper or dash pot.

* Law:

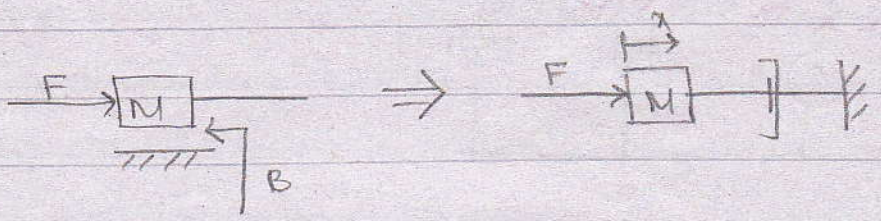
The reactive force on a damper/dashpot is equal to the product of coeff of viscous friction & the relative velocities

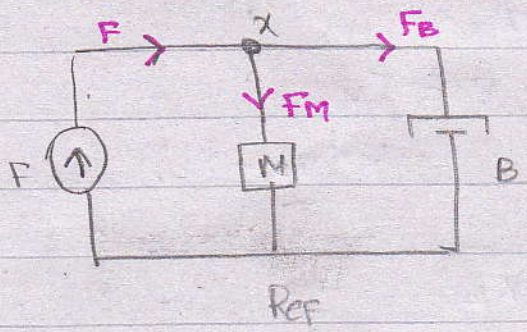
at the end of damper.

$$F = BV$$

B depends on num of holes, viscosity.

Simple Mechanical System





At node x :

$$F = F_M + F_B$$

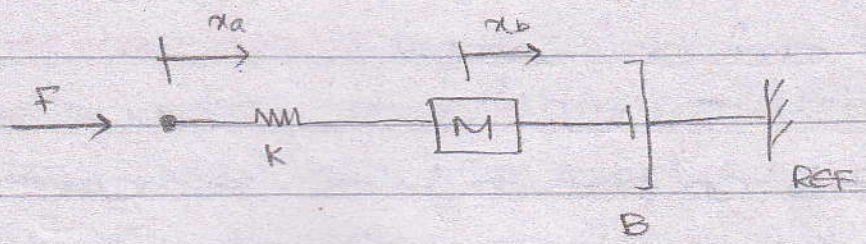
$$F = Ma + BV$$

$$F = MDV + BV \quad \text{OR} \quad = MD^2X + BDX$$

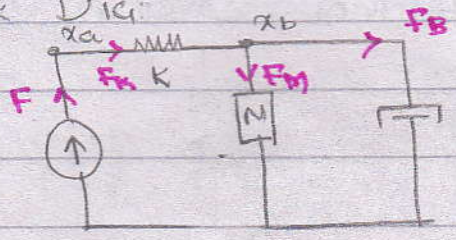
$$F = (MD + B)V \quad \text{OR} \quad = (MD^2 + BD)X$$

OR

$$\Rightarrow \frac{V}{F} = \frac{1}{MD + B} \qquad \frac{X}{F} = \frac{1}{MD^2 + BD}.$$



Network Diagram



At Node x_a

$$F = F_K$$
$$= k(x_a - x_b) \quad \rightarrow (i)$$

At Node x_b

$$F_K = F_M + F_B$$
$$k(x_a - x_b) = M\ddot{x}_b + BV_b \quad \rightarrow (ii)$$
$$= MDV_b + BDx_b$$
$$= MD^2x_b + BDx_b$$
$$Kx_a = MD^2x_b + BDx_b + Kx_b \quad \rightarrow (iii)$$
$$\frac{x_b}{x_a} = \frac{K}{MD^2 + BD + K}$$

Subs (i) in (ii)

$$F = MD^2x_b + BDx_b$$
$$\frac{x_b}{F} = \frac{1}{MD^2 + BD}$$

Subs value of x_b in (iii)

$$kx_a = (MD^2 + BD + k) \times \frac{F}{MD^2 + BD}$$

$$\frac{x_a}{F} = \frac{MD^2 + BD + k}{k(MD^2 + BD)}$$

Proof:

$$\frac{x_a}{F} \times \frac{x_b}{x_a} = \frac{x_b}{F}$$

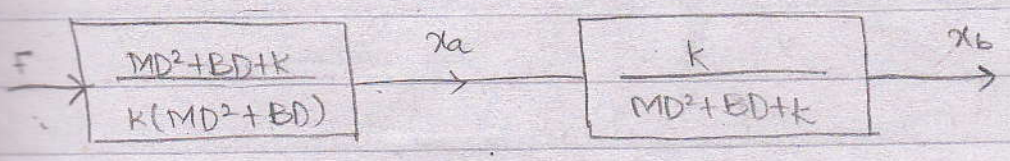
$$\frac{MD^2 + BD + k}{k(MD^2 + BD)} \times \frac{k}{MD^2 + BD + k} = \frac{1}{MD^2 + BD}$$

(ii)

$$\frac{1}{MD^2 + BD} = \frac{1}{MD^2 + BD}$$

→ (iii)

proved!



17th Jan 2014

The State Variable & State Eq

The word state refers to the present, past & future cond's of the sys wch can be described by the set of num of curves & eq's.

'State Variable' is defined as a minimal sets of variables $x_1(t), x_2(t) \dots x_n(t)$ sch that the knowledge of these variables at any time t_0 & inpo on the i/p excitation is sufficient to determine the state of the sys at anytime $t > t_0$.

State Variable of mass spring sys { x, v

wherever it's a mass state variable would be x & v.

$x_1 = x$ $x_2 = v$

Order Of Diff Eq = Num of Stat Variable = Num of Stat Eq's

* State eq is always of 1st Order.

* If the order of DE is 2, we will get 2 state eq of first order

u is the i/p

$$[X'] = [A][X] + [B]u \qquad \text{where } X' = f(x,u)$$

* In mech sys

By diff one S.V we get other SV

ie

$$x \rightarrow \frac{dx}{dt} = V \rightarrow \boxed{x_1 = x_2'}$$

* In Electrical sys

diff won't provide other S.V

ie $\frac{di}{dt} \neq V$

$$x' = f(x, u)$$

$\Downarrow$ $\Downarrow$ i/p

State Variable

$$F = MD^2x + BDx \qquad \rightarrow (1)$$

State Variables

$X_1 = x$ $\qquad$ $X_2 = V$ (for M)

Since Force $F = MD^2X$ input hence $F = u$

$$\cancel{X_2 = MD^2X} \Rightarrow \boxed{X_1' = X_2 + 0u}$$

from ①

$$u = MX_2' + BX_2$$

$$u = MX_2' + BX_2$$

$$\boxed{X_2' = \frac{u}{M} - \frac{BX_2}{M}}$$

$$X_1' = X_2$$

OR

$$\boxed{X_1' = X_2}$$

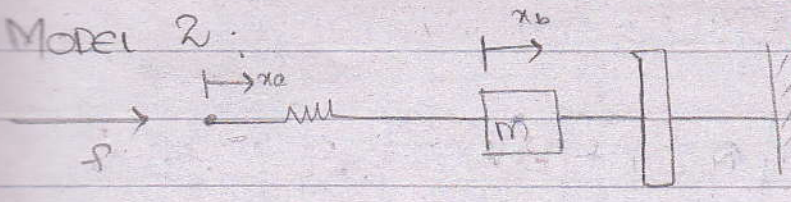
$$X_1' = 0X_1 + 1X_2 + 0u$$

* State Eq Matrix form (S.M.F)

$$\underset{Y'}{\begin{bmatrix} X_1' \\ X_2' \end{bmatrix}} = \underset{A}{\begin{bmatrix} 0 & 1 \\ 0 & -B/M \end{bmatrix}} \underset{X}{\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}} + \underset{B}{\begin{bmatrix} 0 \\ u/M \end{bmatrix}} \underset{u}{u}$$

$$\begin{bmatrix} X_1' \\ X_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -B/M \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M \end{bmatrix} u$$

* Model 2:



CASE 1: Same as previous eg when we consider force as i/p.

CASE 2:-

At node x_a

$$F = K(x_a - x_b)$$

At node x_b.

$$F_K = F_x + F_B \quad (1)$$

$$K(x_a - x_b) = MDV_b + BV_b$$

$$MD^2x_b + BDx_b$$

When $u = x_a$

State Variable:

$$X_1 = x_b$$

$$X_2 = V_b$$

$$X_2 = Dx_b = Dx_1$$

$$\boxed{X_1' = X_2}$$

From (1)

$$Kx_a - Kx_b = MDV_b + BV_b$$

$$Ku - Kx_1 = MX_2' + BX_2$$

$$\Rightarrow X_2' = \frac{Ku}{M} - \frac{Kx_1}{M} - \frac{BX_2}{M}$$

3. M.T

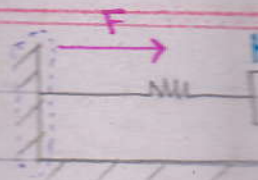
$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -B/M \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ k/m \end{bmatrix} u .$$

OUTPUT EQ

$$[y] = [c][x]$$

$$[y] = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

* Here the Variable x_2 is not req to design a system therefore it's zero.
* Desired O/P or result is always taken as 1



2 Nodes acc

NETWORK M

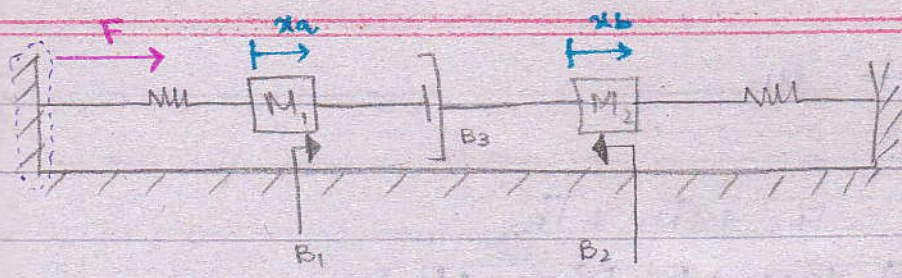
kel at n

F

u = K

u = K

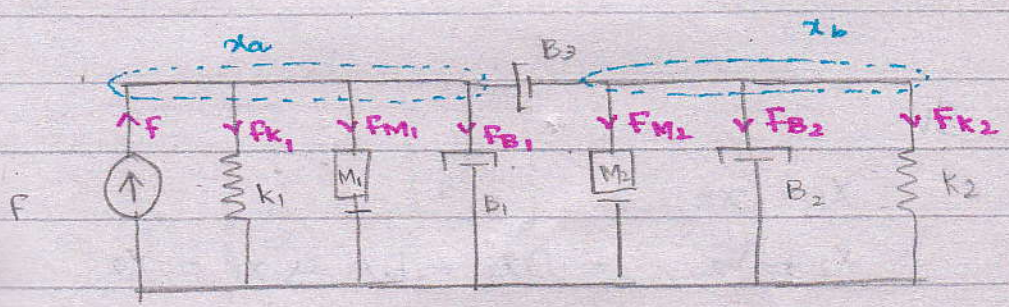
x_2' =



** If reference wall is absent then one side then we have 3 nodes.

2 Nodes across 2 mass

NETWORK MODEL:



KCL at node xa

$$F = F_{k_1} + F_{M_1} + F_{B_1} + F_{B_3}$$

$$= K_1 x_a + M_1 D^2 x_a + B_1 D x_a + B_3 D(x_a - x_b)$$

$$u = K_1 x_a + M_1 D v_a + B_1 v_a + B_3 v_a - B_3 v_b$$

$$u = K_1 X_1 + M_1 X_2' + B_1 X_2 + B_3 X_2 - B_3 X_4$$

$$X_2' = \frac{K X_1}{M_1} + \frac{X_2 (B_1 + B_3)}{M_1} - \frac{B_3 X_4}{M_1} - \frac{u}{M_1}$$

At node x_b

$$F_{B_3} = F_{K_2} + F_{M_2} + F_{B_2}$$

$$B_3 D(x_a - x_b) = K_2 x_b + M_2 D^2 x_b + B_2 D x_b$$

$$B_3 V_a - B_3 V_b = K_2 x_3 + M_2 D V_b + B_2 V_b$$

$$B_3 x_2 - B_3 x_4 = K_2 x_3 + M_2 x_4' + B_2 x_4$$

$$x_4' = \frac{B_3}{M_2} x_2 - \frac{K_2}{M_2} x_3 - \frac{(B_3 + B_2)}{M_2} x_4$$

SV

$$X_1 = x_a$$

$$X_2 = V_a = D x_a = D X_1 \Rightarrow X_1' = X_2$$

$$X_3 = x_b$$

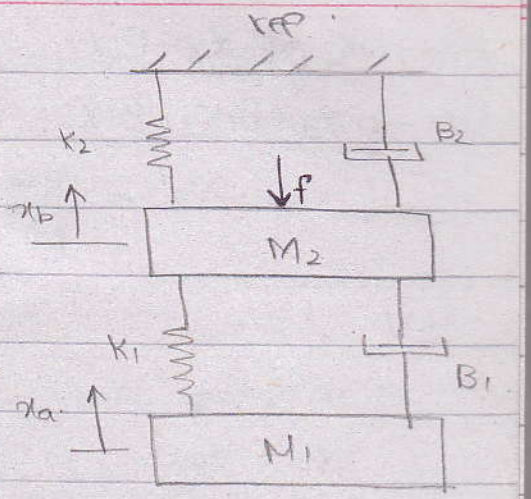
$$X_4 = V_b = D x_b = D X_3 \Rightarrow X_3' = X_4$$

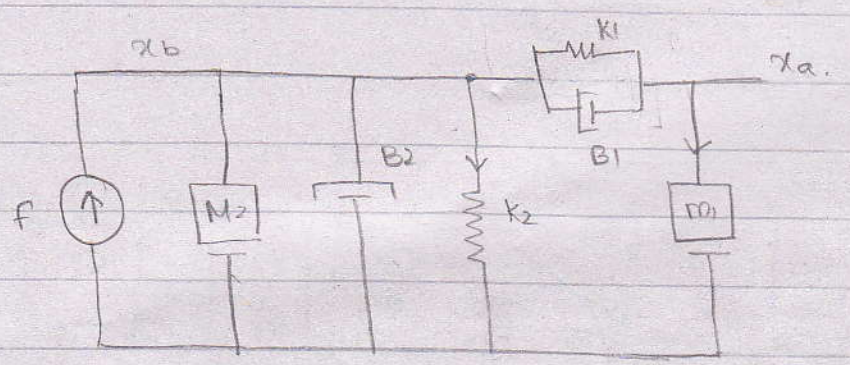
SMF

$$\begin{bmatrix} X_1' \\ X_2' \\ X_3' \\ X_4' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/M_1 & -\frac{(B_1+B_3)}{M_1} & 0 & B_3/M_1 \\ 0 & 0 & 0 & 1 \\ 0 & B_3/M_2 & -\frac{K_2}{M_2} & -\frac{(B_2+B_3)}{M_2} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M_1 \\ 0 \\ 0 \end{bmatrix} u$$

o/p is desired across x_b (assumed)

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$





ref.

KCL at node x_b

$$F = F_{K_2} + B_2 V_b + M_2 D^2 x_b + F_{K_1} + B_1 V_a$$

$$= K_2 x_b + B_2 D x_b + M_2 D^2 x_b + K_1 (x_b - x_a) + B_1 \overbrace{D(x_b - x_a)}$$

$$u = K_2 X_3 + B_2 X_4 + M_2 D X_4 + K_1 X_3 - K_1 X_1 + B_1 X_4 - B_1 X_2$$

$$u = K_2 X_3 + B_2 X_4 + M_2 X_4' + K_1 X_3 - K_1 X_1 + B_1 X_4 - B_1 X_2$$

$$X_4' = \frac{u}{M_2} + \frac{K_1}{M_2} X_1 + \frac{B_1}{M_2} X_2 - \frac{(K_2 + K_1)}{M_2} X_3 + \frac{(B_1 + B_2)}{M_2} X_4$$

KCL @ node x_a

$$F_{K_1} + F_{B_1} = M_1 D^2 x_a$$

$$K_1 (x_a - x_b) + B_1 (D x_a - D x_b) = M_1 D^2 x_a$$

$$K_1x_1 - K_1x_3 + B_1x_4 - B_1x_2 = M_1 D x_2$$

$$K_1x_1 - K_1x_3 + B_1x_4 - B_1x_2 = M_1 x_2'$$

$$x_2' = \frac{K_1}{M_1}x_1 - \frac{B_1}{M_1}x_2 - \frac{K_1}{M_1}x_3 + \frac{B_1}{M_1}x_4$$

SV

$$x_1 = x_a$$

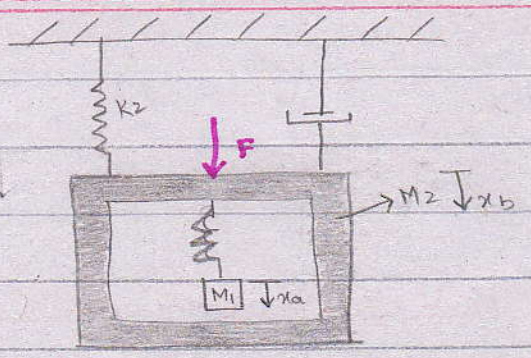
$$x_2 = V_a = D x_a = D x_1 \Rightarrow x_1' = x_2$$

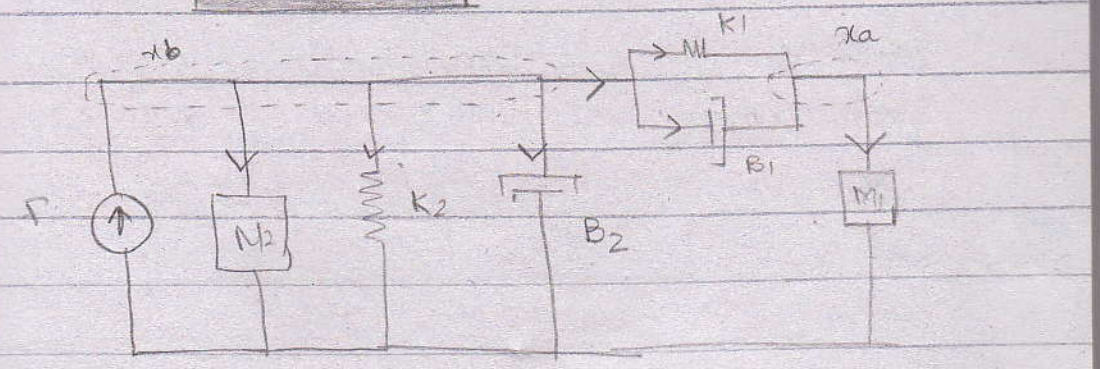
$$x_3 = x_b$$

$$x_4 = V_b = D x_b = D x_3 \Rightarrow x_3' = x_4$$

SMF

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ K_1/M_1 & -B_1/M_1 & -K_1/M_1 & B_1/M_1 \\ 0 & 0 & 0 & 1 \\ K_1/M_2 & B_1/M_2 & -(K_1+K_2)/M_2 & (B_1+B_2)/M_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M_2 \\ 0 \\ 0 \end{bmatrix} u$$





At node x_b

$$F = F_{M_2} + F_{K_2} + F_{B_2} + F_{K_1} + F_{B_1}$$

$$= M_2 D^2 x_b + K_2 x_b + B_2 D x_b + K_1 (x_b - x_a) + B_1 D (x_b - x_a)$$

$$u = M_2 D X_4 + K_2 X_3 + B_2 X_4 + K_1 X_3 - K_1 X_1 + B_1 X_4 - B_1 D X_1$$

$$= M_2 X_4' + (K_2 + K_1) X_3 + (B_1 + B_2) X_4 - K_1 X_1 - B_1 X_2$$

$$X_4' = \frac{u}{M_2} - \frac{(K_2 + K_1) X_3}{M_2} - \frac{(B_1 + B_2) X_4}{M_2} + \frac{K_1 X_1}{M_2} + \frac{B_1 X_2}{M_2}$$

At node x_a

$$K_1 (x_b - x_a) + B_1 D (x_b - x_a) = M_1 D^2 x_a$$

$$(K_1 + B_1 D) x_b - (K_1 + B_1 D) x_a = M_1 D^2 x_a$$

$$K_1 x_b + B_1 D x_b - K_1 x_a - B_1 D x_a = M_1 D^2 x_a$$

$$K_1 X_3 + B_1 X_4 - K_1 X_1 - B_1 X_2 = M_1 D X_2$$

$$X_2' = \frac{-K_1 X_1}{M_1} - \frac{B_1 X_2}{M_1} + \frac{K_1 X_3}{M_1} + \frac{B_1 X_4}{M_1}$$

3.V

$x_1 = x_a$

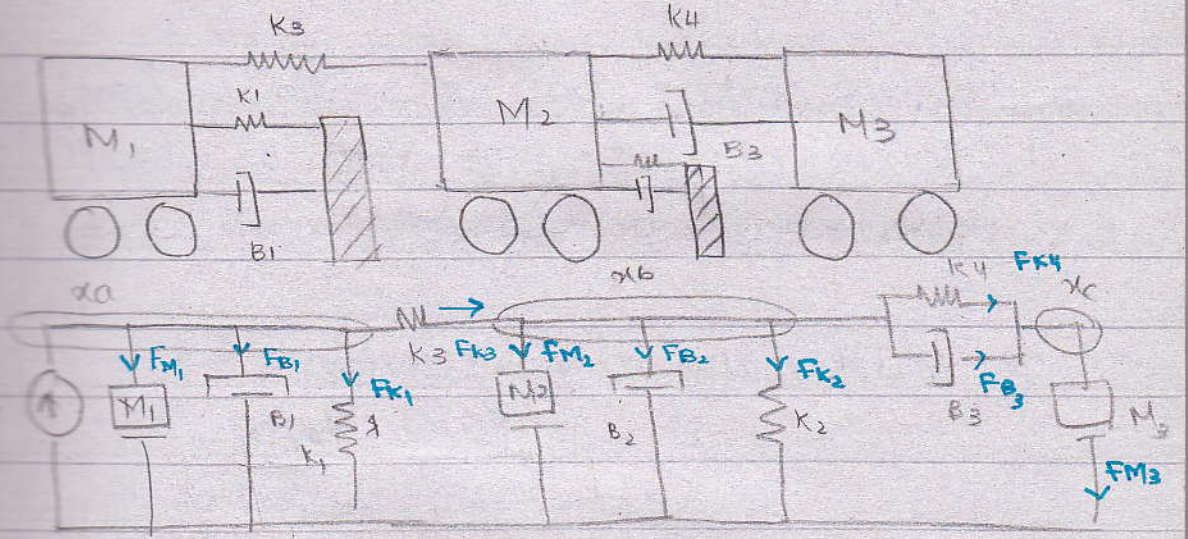
$x_2 = v_a = Dx_a = Dx_1 \Rightarrow x_1' = x_2$

$x_3 = x_b$

$x_4 = v_b = Dx_b = Dx_3 \Rightarrow x_3' = x_4$

Now SMF is

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K_1/M_1 & -B_1/M_1 & K_1/M_1 & +B_1/M_1 \\ 0 & 0 & 0 & 1 \\ K_1/M_2 & B_1/M_2 & -(K_1+K_2)/M_2 & -(B_1+B_2)/M_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/M_2 \end{bmatrix}$$



KCL @ node xa

$$F = F_{M_1} + F_{B_1} + F_{K_1} + F_{K_3}$$

$$U = M_1 D^2 x_a + B_1 D x_a + k_1 x_a + k_3 (x_a - x_b)$$

$$U = M_1 x_2' + B_1 x_2 + k_1 x_1 + k_3 x_1 - k_3 x_3$$

$$x_2' = \frac{U}{M_1} - \frac{(K_1 + K_3)}{M_1} x_1 - \frac{B_1}{M_1} x_2 + \frac{K_3}{M_1} x_3$$

KCL @ node xb

$$F_{K_3} = F_{M_2} + F_{B_2} + F_{K_2} + F_{K_4} + F_{B_3}$$

$$k_3 (x_a - x_b) = M_2 D^2 x_b + B_2 D x_b + k_2 x_b + k_4 (x_b - x_c) + B_3 D (x_b - x_c)$$

$$k_3 x_1 - k_3 x_3 = M_2 x_4' + B_2 x_4 + k_2 x_3 + k_4 x_3 - k_4 x_5 + B_3 x_4 - B_3 x_6$$

$$x_4' = \frac{k_3}{M_2} x_1 - \frac{(k_3 + k_2 + k_4)}{M_2} x_3 - \frac{(B_2 + B_3)}{M_2} x_4 + \frac{k_4}{M_2} x_5 + \frac{B_3}{M_2} x_6$$

KCL @ node x_c:

$$f_{k4} + f_{B3} = F_{m3}$$

$$k_4(x_b - x_c) + B_3 D(x_b - x_c) = M_3 D^2 x_c$$

$$k_4 x_3 - k_4 x_5 + B_3 x_4 - B_3 x_6 = M_3 x_6'$$

$$x_6' = \frac{k_4 x_3}{M_3} + \frac{B_3 x_4}{M_3} - \frac{k_4 x_5}{M_3} - \frac{B_3 x_6}{M_3}$$

S.V

$x_1 = x_a$

$x_2 = V_a = D x_a = D x_1 = x_1' = x_2$

$x_3 = x_b$

$x_4 = V_b = D x_b = D x_3 = x_3' = x_4$

$x_5 = x_c$

$x_6 = V_c = D x_c = D x_5 = x_5' = x_6$

S.M.F

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ x_5' \\ x_6' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{(k_1+k_3)}{M_1} & -B_1/M_1 & k_3/M_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ k_3/M_2 & 0 & \frac{(k_2+k_3+k_4)}{M_2} & \frac{(B_2+B_3)}{M_2} & \frac{k_4}{M_2} & \frac{B_3}{M_2} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & k_4/M_3 & \frac{B_3}{M_3} & -\frac{k_4}{M_3} & -\frac{B_3}{M_3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Mechanical Rotational System (MRS)

(MRS) are those in wch motion takes place about a fix point (abt an axis)
The Elements of an interest are

→ The moment of Inertia (J) ← mass
→ Damper (B)
→ Torsional Spring (K)

The variables of interest are

→ Angular displacement (θ)
→ " Velocity (ω)
→ " acceleration (α)

Elements	Energy	Torque
Inertia (J)	$\frac{1}{2} J\omega^2$	$\tau = J\alpha$
Damper (B)	—	$\tau = B\omega$
Torsional Spring (K)	$\frac{1}{2} K\theta^2$	$\tau = K\theta$

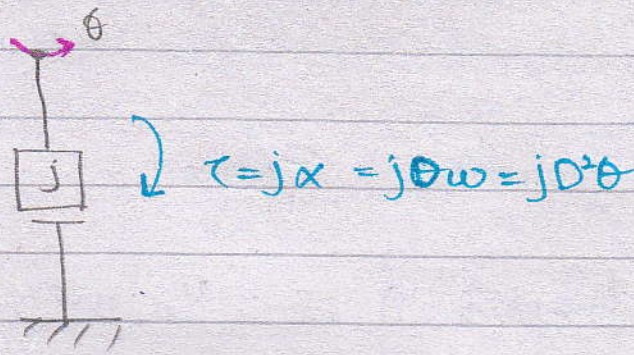
* Dynamics of a sys remains same even if the dir of force is reversed.

Sum of all torques acting on a rest point or ~~axis~~ about an axis is equals to zero or.

Applied torque = Reactive torque

Moment Of Inertia

Moment of inertia is is the element wch stores the KE of the rot. motion. It depends on the mass of the body & the distribution of mass wrt the axis of rot.

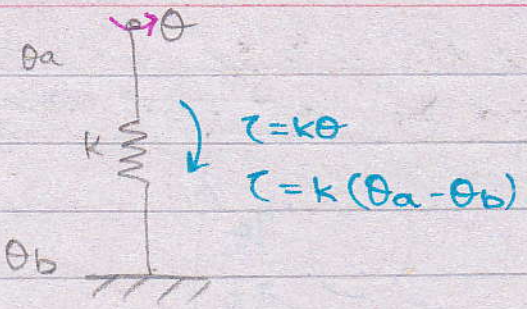


Torsional Spring

It may be an actual Spring a cable or a rod that characterizes of an element wch stores the PE & tht it's mag will $\frac{1}{2}k\theta^2$

* Damper:
Damping moves of the

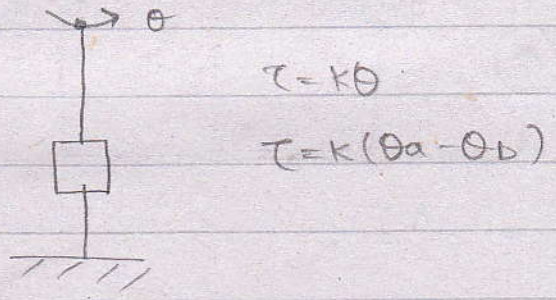
*Rotation as

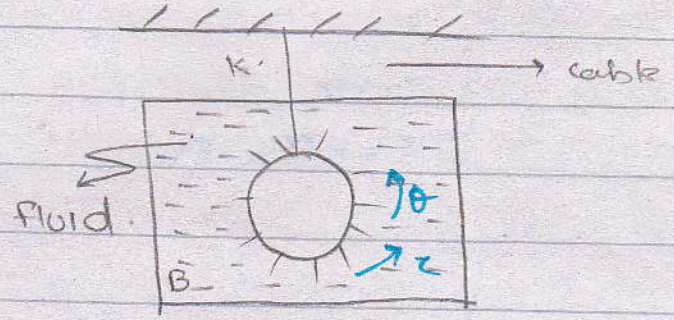


* DAMPER:

Damping occurs whenever a body moves through a fluid. It characterizes of the element that absorbs energy

*Rotation sys are generally represented as

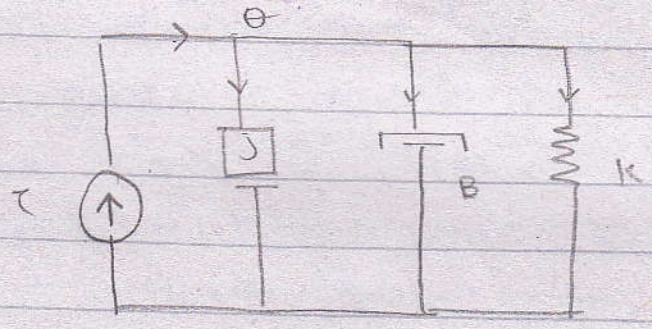




mass ↶ torque ⇓

moment of inertia will produce.

* We need to make it's mech model first



Ref.

at node θ

$$\tau = \tau_J + \tau_B + \tau_K$$

$$= J\alpha + B\omega + k\theta$$

$$= JD\omega + B\omega + k\theta$$

$$= JD^2\theta + BD\theta + k\theta$$

$$\boxed{\frac{\theta}{\tau} = \frac{1}{JD^2 + BD + K}}$$

S-V

$$x_1 = \theta$$

$$x_2 = \omega = D\theta = Dx_1 = x_1' \Rightarrow x_1' = x_2$$

$$u = jDx_2 + Bx_2 + Kx_1$$

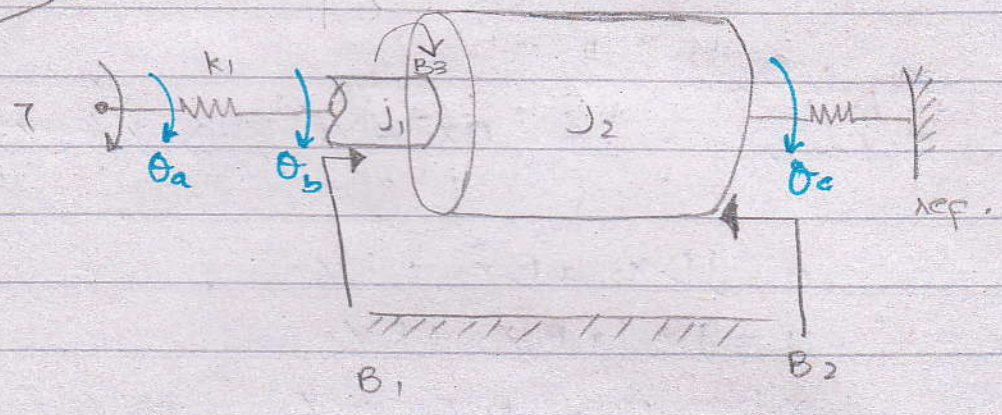
$$u = jx_2' + Bx_2 + Kx_1$$

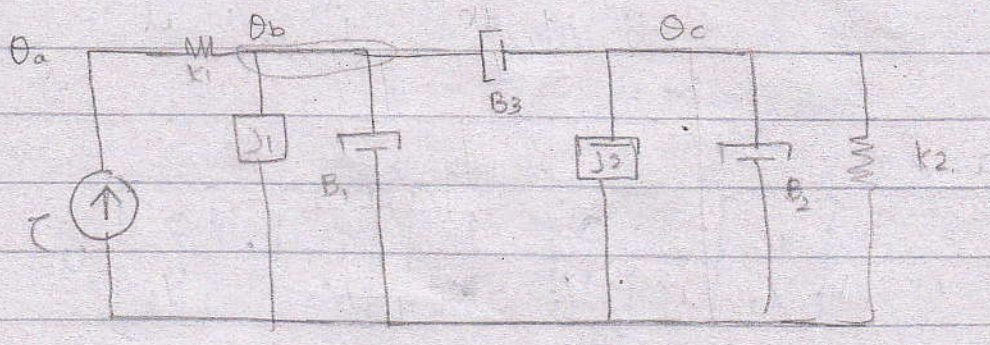
$$x_2' = \frac{u}{j} - \frac{B}{j}x_2 - \frac{K}{j}x_1$$

S.M.F

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K/j & -B/j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/j \end{bmatrix} u$$

MODEL 2





At θ_a

$$\tau = \tau_{K1}$$

$$\tau = K_1(\theta_a - \theta_b) \quad \rightarrow (1)$$

At θ_b

$$\tau_{K1} = \tau_{J1} + \tau_{B1} + \tau_{B3}$$

$$K_1(\theta_a - \theta_b) = J_1\alpha + B_1\omega + B_3\omega$$

$$K_1(\theta_a - \theta_b) = J_1 D^2\theta_b + B_1 D\theta_b + B_3 D(\theta_b - \theta_c)$$

$$= J_1 D^2\theta_b + B_1 D\theta_b + B_3 D\theta_b - B_3 D\theta_c \quad \rightarrow (2)$$

At θ_c

$$\tau_{B3} = \tau_{J2} + \tau_{B2} + \tau_{K2}$$

$$B_3 D(\theta_b - \theta_c) = J_2 \alpha + B_2 D\theta_c + K_2 \theta_c$$

$$B_3 D\theta_b - B_3 D\theta_c = J_2 D^2\theta_c + B_2 D\theta_c + K_2\theta_c \quad \rightarrow (3)$$

State Variables

$x_1 = \theta_b$

$x_2 = V_b = D\theta_b = Dx_1 = x_1' = x_2$

$x_3 = \theta_c$

$x_4 = V_c = D\theta_c = Dx_3 = x_3' = x_4$

here we have 2 inputs

1 $\Rightarrow \tau = U$

2 $\Rightarrow \theta_a = U$

Case 1

$$(2) \Rightarrow \tau = U = J_1 DV_b + B_1 x_2 + B_3 x_2 - B_3 x_4$$

from (1)

$$= J_1 x_2' + B_1 x_2 + B_3 x_2 - B_3 x_4$$

$$U = J_1 x_2' + x_2(B_1 + B_3) - B_3 x_4$$

$$x_2' = \frac{U}{J_1} + \frac{B_3 x_4}{J_1} + \frac{(B_1 + B_3)x_2}{J_1}$$

③ $\Rightarrow B_3X_2 - B_3X_4 = J_2DX_4 + B_2X_4 + K_2X_3$

$B_3X_2 - B_3X_4 = J_2X_4' + B_2X_4 + K_2X_3$

$$X_4' = \frac{B_3X_2}{J_2} - \frac{(B_3+B_2)X_4}{J_2} - \frac{K_2X_3}{J_2}$$

NOW, SMF

$$\begin{bmatrix} X_1' \\ X_2' \\ X_3' \\ X_4' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{B_1+B_3}{J_1} & 0 & B_3/J_1 \\ 0 & 0 & 0 & 1 \\ 0 & B_3/J_2 & \frac{-K_2}{J_2} & \frac{-(B_2+B_3)}{J_2} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J_1 \\ 0 \\ 0 \end{bmatrix} U$$

CASE II:

② $\Rightarrow K_1U - K_1\theta_b = J_1D^2\theta_b + B_1D\theta_b + B_3D\theta_b - B_3D\theta_2$

$K_1U = J_1DX_2 + B_1X_2 + B_3X_2 - B_3X_4 + K_1X_1$

$K_1U = J_1X_2' + (B_1+B_3)X_2 + K_1X_1 - B_3X_4$

$$X_2' = \frac{K_1U}{J_1} - \frac{K_1X_1}{J_1} - \frac{(B_1+B_3)X_2}{J_1} + \frac{B_3X_4}{J_1}$$

③ $\Rightarrow B_3 X_2 - B_3 X_4 = J_2 DX_4 + B_2 X_4 + K_2 X_3$

$B_3 X_2 - B_3 X_4 = J_2 X_4' + B_2 X_4 + K_2 X_3$

$$X_4' = \frac{B_3}{J_2} X_2 - \frac{K_2}{J_2} X_3 - \frac{(B_2 + B_3)}{J_2} X_4$$

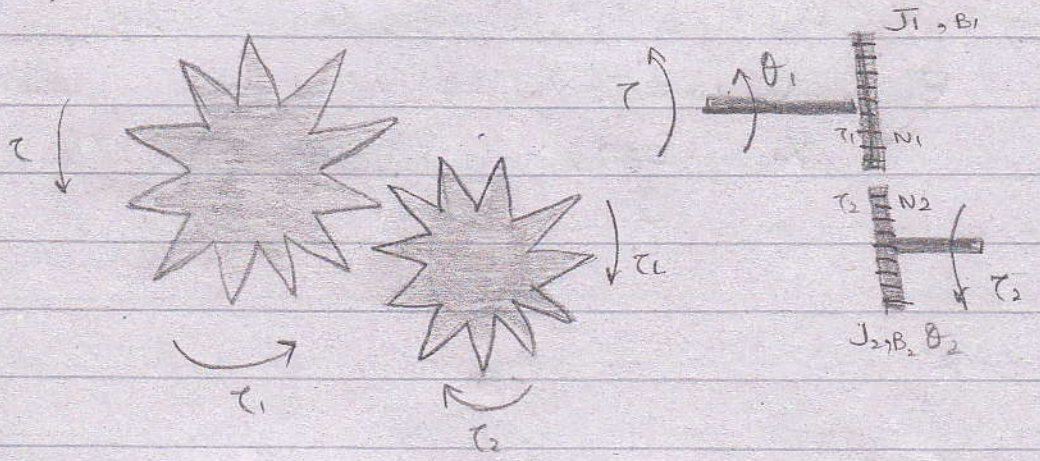
SMF

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_1}{J_1} & -\frac{(B_1+B_3)}{J_1} & 0 & B_3/J_1 \\ 0 & 0 & 0 & 1 \\ 0 & B_3/J_2 & -\frac{K_2}{J_2} & -\frac{(B_2+B_3)}{J_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} +$$

$$u \begin{bmatrix} 0 \\ K_1/J_1 \\ 0 \\ 0 \end{bmatrix}$$

27th JAN 2014

GEAR TRAIN



Gear train is a mech device that is used to transfer the mech power from 1 part 2 another. The device is also known as matching device.

* This device works similar to T/F
* Used for τ scaling

N = no of teeth

Gear train dynamics & power transfer theory can provide us the following facts.

- The no of teeths on each gear is directly prop to the diameter of gear (ie radius)

$r_1 \propto N_1$ $r_2 \propto N_2$

$$\frac{r_1}{r_2} = \frac{N_1}{N_2}$$

where
r = radius

- The distance covered on the surface of each gear is same

$$s_1 = s_2$$
$$r_1\theta_1 = r_2\theta_2$$
$$\Rightarrow \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1}$$

- The workdone by each gear is same assuming no loss.

$$W_1 = W_2 \qquad \because W = \tau\theta$$
$$\tau_1\theta_1 = \tau_2\theta_2$$
$$\Rightarrow \frac{\tau_1}{\tau_2} = \frac{\theta_2}{\theta_1}$$

- The power of each gear is also same

$$P_1 = P_2$$
$$\tau_1\omega_1 = \tau_2\omega_2$$
$$\Rightarrow \frac{\tau_1}{\tau_2} = \frac{\omega_2}{\omega_1}$$

GEAR RATIO:

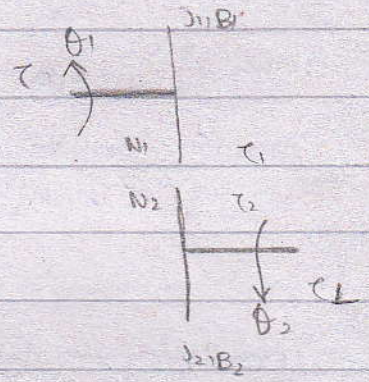
→ Rep by n_1

Ratio of the speed of driving shaft (ω_1) & the speed of driven shaft (ω_2)

$$\eta_a = \frac{\omega_1}{\omega_2} \quad \text{driving shaft} \quad \text{driven shaft}$$

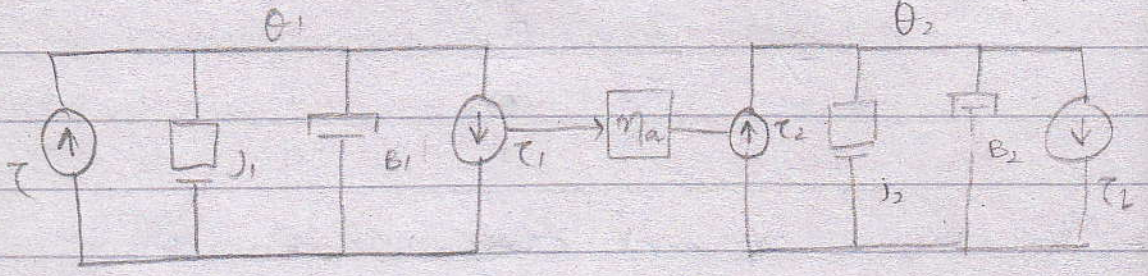
$$\eta_a = \frac{\theta_1}{\theta_2} = \frac{r_2}{r_1} = \frac{N_2}{N_1} = \frac{\tau_2}{\tau_1}$$

Model:



NETWORK MODEL

FIRST GEAR



driving driven

KCL @ node θ1

$$\tau = J_1 D^2\theta_1 + B_1 D\theta_1 + \tau_1 \quad \rightarrow (1)$$

At node θ_2

$$\tau_2 = J_2 D^2\theta_2 + B_2 D\theta_2 + \tau_L \quad \rightarrow (2)$$

$$\tau_2 = J_2 D^2\theta_2 + B_2 D\theta_2$$

Now τ as as funct of θ_1 $\quad \tau = f(\theta_1)$

CASE 1:

$$(1) \Rightarrow \tau = J_1 D^2\theta_1 + B_1 D\theta_1 + \tau_1$$

$$\tau - \tau_1 = J_1 D^2\theta_1 + B_1 D\theta_1$$

$$\because \quad n_a = \frac{\tau_2}{\tau_1}$$

$$\Rightarrow \tau - \frac{\tau_2}{n_a} = J_1 D^2\theta_1 + B_1 D\theta_1$$

$$\tau = J_1 D^2\theta_1 + B_1 D\theta_1 + \frac{1}{n_a}\left(\tau_2\right) \rightarrow (3)$$

$$(2) \Rightarrow \tau_2 = J_2 D^2\theta_2 + B_2 D\theta_2 + \tau_L$$

$$= J_2 D^2\frac{\theta_1}{n_a} + B_2 D\frac{\theta_1}{n_a} + \tau_L$$

$$= \frac{1}{n_a}\left[J_2 D^2\theta_1 + B_2 D\theta_1\right] + \tau_L$$

Subs in Eq (3)

$$(3) \Rightarrow \tau = J_1 D^2\theta_1 + B_1 D\theta_1 + \frac{1}{n_a^2}\left[J_2 D^2\theta_1 + B_2 D\theta_1\right] + \tau_L$$

$$\Rightarrow \tau = \left[J_1 + \frac{1}{n a^2} J_2\right] D^2\theta_1 + \left[B_1 + \frac{B_2}{n a_2^2}\right] D\theta_1 + \frac{\tau_L}{n a}$$

$\Downarrow$ J_{eq} $\Downarrow$ B_{eq}

$$\tau - \frac{\tau_L}{n a} = J_{eq} D^2\theta_1 + B_{eq} D\theta_1$$

$$\tau = J_{eq} D^2\theta_1 + B_{eq} D\theta_1$$

here

τ_L represents any other load (corrected)

But ideally we Assume

no load Condition

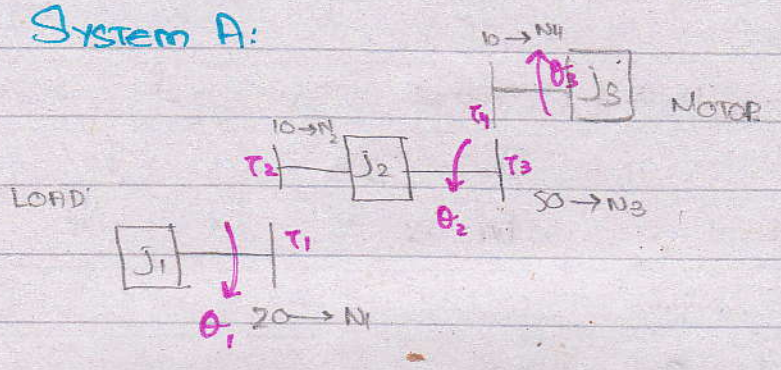
Therefore τ_L is taken 2 be zero

31st Jan 2014.

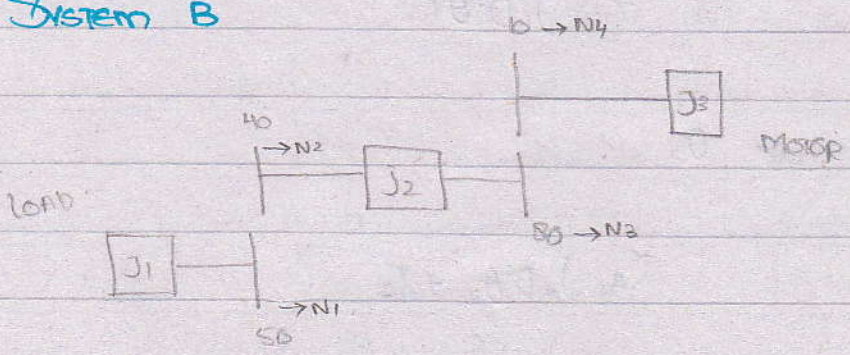
The 2 gear trains as shown in Fig have an identical net reduction, identical inertias at each stage. 3 are driven by the same motor. The no of teeths on each gear is indicated on the figure @ the instant of starting the motor develops a torque 'τ' wch system has higher initial load & acceleration.

1

System A:



System B

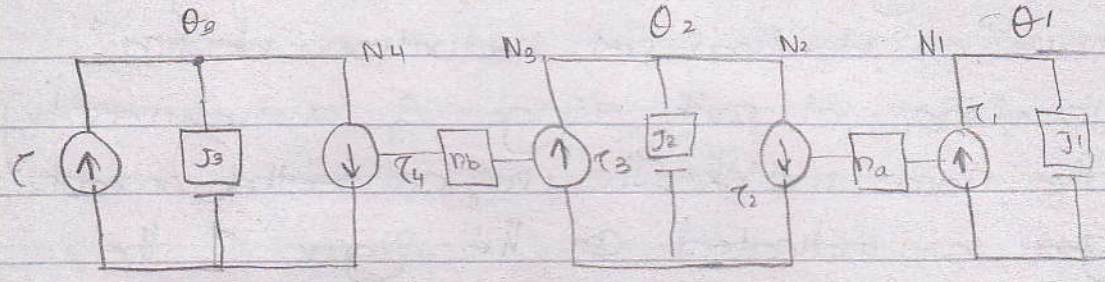


When

a) $J_1 = J_2 = J_3$

b) $J_1 = 40 J_2 = 4 J_3$

drive/driven



$$\eta = \frac{\omega_1}{\omega_2} = \frac{\theta_1}{\theta_2} = \frac{\tau_2}{\tau_1} = \frac{N_2}{N_1}$$

$$\Rightarrow \quad \eta_b = \frac{\theta_3}{\theta_2} = \frac{\omega_3}{\omega_2} = \frac{\tau_3}{\tau_2} = \frac{N_3}{N_4}$$

$$\Rightarrow \quad \eta_a = \frac{\theta_2}{\theta_1} = \frac{\omega_2}{\omega_1} = \frac{\tau_1}{\tau_2} = \frac{N_1}{N_2}$$

At node θ_1

$$\tau_1 = J_1 D^2 \theta_1$$

At node θ_2

$$\tau_3 = J_2 D^2 \theta_2 + \tau_2$$

$$= J_2 D^2 \theta_2 + \frac{\theta_1}{\theta_2} \tau_1$$

$$= J_2 D^2 \theta_2 + \frac{\theta_1}{\theta_2} (J_1 D^2 \theta_1)$$

At node

At node θ_3

$$\tau = J_3 D^2\theta_3 + \tau_4$$

$$\tau = J_3 D^2\theta_3 + \frac{\tau_3 \theta_2}{\theta_3}$$

$$\tau = J_3 D^2\theta_3 + \frac{\theta_2}{\theta_3}\left\{ J_2 D^2\theta_2 + \frac{\theta_1}{\theta_2}(J_1 D^2\theta_1)\right\}$$

$$\tau = J_3 D^2\theta_3 + \frac{\theta_2}{\theta_3} J_2 D^2\theta_2 + \frac{\theta_1}{\theta_2} J_1 D^2\theta_1$$

$$= J_3 D^2\theta_2 n_b + \frac{\theta_2}{\theta_3} J_2 D^2\theta_1 n_a + \frac{\theta_1}{\theta_3} J_1 D^2\theta_1$$

$$= J_3 D^2\theta_1 n_a n_b + \frac{\theta^2}{\theta_3} J_2 D^2\theta_1 n_a + \frac{\theta_1}{\theta_3} J_1 D^2\theta_1$$

$$= D^2\theta_1\left\{ J_3 n_a n_b + \frac{\theta_2}{\theta_3} J_2 n_a + \frac{\theta_1}{\theta_3} J_1\right\}$$

$$= D^2\theta_1\left\{ J_3 \frac{N_1}{N_2}\frac{N_3}{N_4} + \frac{\theta_2}{\theta_3} J_2 \frac{N_1}{N_2} + \frac{\theta_1}{\theta_3} J_1\right\}$$

$$= D^2\theta_1\left\{ \frac{N_1 N_3}{N_2 N_4} J_3 + \frac{N_1 N_4}{N_2 N_3} J_2 + \frac{\theta_2}{\theta_3 n_a} J_1\right\}$$

$$= D^2\theta_1\left\{ \frac{N_1 N_3}{N_2 N_4} J_3 + \frac{N_1 N_4}{N_2 N_3} J_2 + \frac{N_4 N_2}{N_3 N_1} J_1\right\}$$

CASE (a)

When $J_1 = J_2 = J_3$

$$\tau = \left\{ \frac{N_1 N_3}{N_2 N_4} + \frac{N_1 N_4}{N_2 N_3} + \frac{N_2 N_4}{N_1 N_3} \right\} J\alpha_1$$

* For Sys A:-

$$\tau_A = \left\{ \frac{20 \times 50}{10 \times 10} + \frac{20 \times 10}{10 \times 50} + \frac{10 \times 10}{20 \times 50} \right\} J\alpha_1$$

$$= 10.5 \; J\alpha_1$$

* For Sys B:-

$$\tau_B = \left\{ \frac{50 \times 80}{40 \times 10} + \frac{50 \times 10}{80 \times 40} + \frac{40 \times 10}{50 \times 80} \right\} J\alpha_2$$

$$= 10.256 J\alpha_2$$

Since both the systems operates with same motor

$$\tau_A = \tau_B$$

$$10.5 J\alpha_1 = 10.256 J\alpha_2$$

$$\boxed{\alpha_1 = 0.977 \alpha_2}$$

Case (b)

When $J_1 = 40 J_2 = 4 J_3$

$$T = \left[\frac{N_1N_3}{N_2N_4} J_3 + \frac{N_1N_4}{N_2N_3} J_2 + \frac{N_2N_4}{N_1N_3} J_1\right] D^2\theta_1$$

$$= \left[\frac{20\times50}{10\times10} \frac{J_1}{4} + \frac{20\times10}{10\times50} \frac{J_1}{40} + \frac{10\times10}{20\times50} J_1\right] D^2\theta_1$$

$$T_B = \left\{\frac{50\times80}{40\times10} + \frac{50\times10(40)^{-1}}{80\times40} \frac{40\times10(40)^{-2}}{50\times80}\right\} J\alpha_2$$

$$= 10.03\,\alpha_2 J_1$$

$$T_A = T_B$$

$$10.035\, J_1\alpha_1 = 10.03\, J_1\alpha_2$$

$$\alpha_1 = 0.99\,\alpha_2$$

Electrical Systems

Elements	Energy	Voltage / Current
1) Resistor	—	$V=IR \Rightarrow I = V/R$
2) Inductor	$\frac{1}{2} Li^2$	$V_L = LDi$ $\Rightarrow i_L = V_L/LD$
3) Capacitor	$\frac{1}{2} CV^2$	$i_C = CDV$ $V_C = \frac{1}{LD} i_C$

*KCL

- Sum of all voltages across a loop is zero

OR

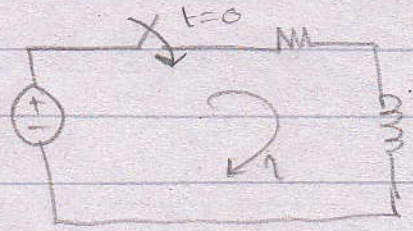
- Input v = o/p v

OR

- Sum of current entering a ~~loop~~ node is equals to the sum of i leaving a node.

In loop equations state variable will be I provided that there's a inductor.

ckt 1



Applying kvl:

$$e = V_R + V_L$$
$$e = iR + LDi$$
$$e = i(R + LD)$$
$$\boxed{\frac{i}{e} = \frac{1}{R+LD}}$$

and

for $\frac{V_R}{e}$ . . . . . .

Applying kvl

$$e = iR + LD\frac{V_R}{R} \qquad \because i = \frac{V_R}{R}$$

$$e = V_R\left(1 + \frac{LD}{R}\right)$$

$$\boxed{\frac{V_R}{e} = \frac{1}{1 + LD/R}}$$

For $\frac{V_L}{e}$

$$i_R = i_L \qquad V_L = LDi$$
$$i_R = \frac{V_L}{LD}$$

$e = V_R + V_L$
$V_R = e - V_L$
$iR = e - V_L$

$$\frac{e - V_L}{R} = \frac{V_L}{LD}$$

$$\frac{e}{R} - \frac{V_L}{R} = \frac{V_L}{LD}$$

$$\frac{e}{R} = \frac{V_L}{R} + \frac{V_L}{LD} = \frac{V_L(LD+R)}{RLD}$$

$$e = V_L\frac{(LD+R)}{LD} \Rightarrow \frac{V_L}{e} = \frac{LD}{LD+R}$$

$$\boxed{\frac{V_L}{e} = \frac{1}{1+R/LD}}$$

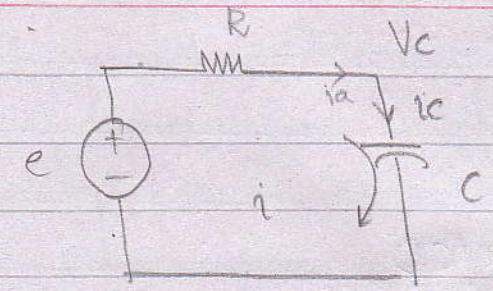
STATE VARIABLES:

$$x = i$$
$$u = e$$

$$u = xR + Lx'$$

$$x' = \frac{-R}{L}x + \frac{u}{L}$$

ckt #2



$\frac{V_c}{e}$

* For i Use KVL
* For V Use KCL

Applying KCL

$i_R = i_C$

$\frac{e - V_c}{R} = CDV_c$

$\frac{e}{R} - \frac{V_c}{R} = CDV_c$

$\frac{e}{R} = \frac{V_c}{R} + CDV_c$

$\frac{e}{R} = V_c\left(\frac{1}{R} + CD\right)$

$= \frac{V_c}{R}(1 + CDR)$

$e = V_c(1 + CDR)$

$$\Rightarrow \boxed{\frac{V_c}{e} = \frac{1}{1 + CDR}} \quad (1)$$

* State variable can be the o/p var but o/p variable wont.

* $\frac{i}{e}$, $\frac{V_R}{e}$.

Applying KVL

$$e = V_R + V_C$$

$$e = iR + i/CD$$

$$e = i\left(R + \frac{1}{CD}\right)$$

$$\boxed{\frac{i}{e} = \frac{1}{R + 1/CD}}$$

Using eqn ①

$$e = V_R + V_c$$

$$e = V_R + \frac{e}{1+CDR} \quad \text{from ①}$$

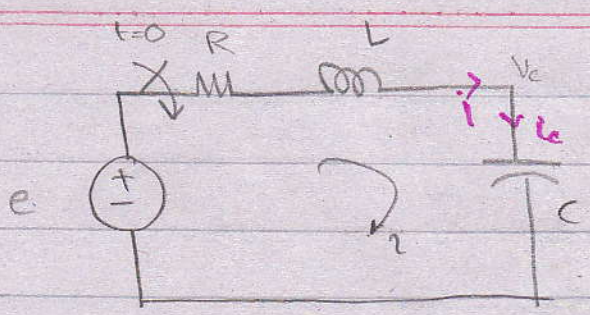
$$V_R = e - \frac{e}{1+CDR}$$

$$V_R = e\left(1 - \frac{1}{1+CDR}\right)$$

$$\frac{V_R}{e} = \frac{1+CDR-1}{1+CDR} = \frac{CDR}{1+CDR}$$

$$\boxed{\frac{V_R}{e} = \frac{1}{1/CDR + 1}}$$

Ckt #3



$\frac{i}{e}$, $\frac{V_c}{e}$

$\frac{i}{e} = ?$ $\quad \frac{V_c}{e} = ?$ $\quad \frac{V_R}{e} = ?$

KVL

$\Rightarrow e = V_R + V_L + V_C$ $\quad \rightarrow ①$

$i = C\frac{dV}{}$

$= iR + LD\frac{V_R}{R} + \frac{1}{CD} i$

$= iR + LDi + \frac{i}{CD}$

$= i(R + LD + 1/CD)$

$\frac{i}{e} = \frac{1}{R + LD + 1/CD}$

$\frac{i}{e} = \frac{CD}{RCD + LCD^2 + 1}$

$e = iR + LDi + V_c$

$e - V_c = i(R + LD)$

$i = \frac{e - V_c}{R + LD}$

* STATE VARIABLES:

$e = u$

$x_1 = i$

$x_2 = V_c$

then

$\textcircled{1} \Rightarrow e = V_R + V_L + x_2$

$= iR + LDi + x_2$

$u = x_1 R + x_1' L + x_2$

$\Rightarrow x_1' = \frac{u}{L} - \frac{x_1 R}{L} - \frac{x_2}{L}$

Now,

applying KCL

$i = i_c$

$i = CDV_c$

$x_1 = C x_2'$

$\Rightarrow x_2' = \frac{x_1}{C}$

Now,

SMF

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -R/L & -1/L \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + u \begin{bmatrix} 1/L \\ 0 \end{bmatrix}$$

ckt #4

Sol 1

Sol 2

$\frac{1}{CD}$

$\frac{1}{C}$

Sol ii

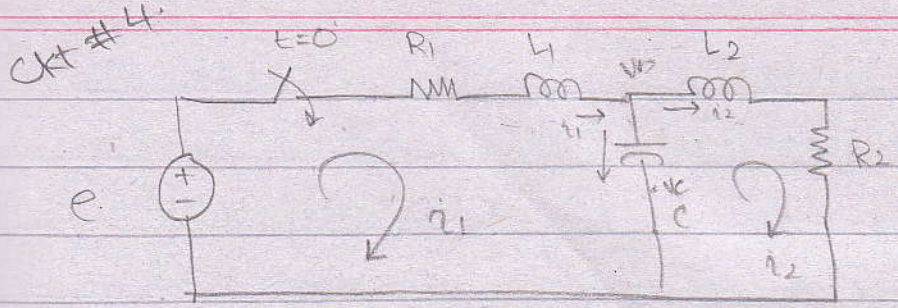
$\frac{1}{CD}$

$\frac{e}{CD(R_1+L_1D+1/CD}$

$\frac{e}{CD(R_1+L_1D+1/C}$

$\frac{q_2}{e}$ =

Ckt #4



$i_1 = i_2 + i_{1-2}$

Eq 1

$$e = V_R + V_L + V_C$$

$$e = iR_1 + L_1 D i_1 + \frac{1}{CD}(i_1 - i_2)$$

$$e = (R_1 + L_1 D + 1/CD) i_1 - 1/CD\, i_2 \rightarrow (1)$$

Eq 2

$$V_C = V_{L_2} + V_{R_2}$$

$$\frac{1}{CD}(i_1 - i_2) = L_2 D i_2 + R_2 i_2$$

$$\frac{1}{CD}(i_1) - \frac{1}{CD} i_2 = L_2 D i_2 + R_2 i_2 \rightarrow 2$$

Subs i1

$$\frac{1}{CD}\left[\frac{e + 1/CD}{R_1 + L_1 D + 1/CD}\right] = \frac{1}{CD} i_2 + L_2 D i_2 + R_2 i_2$$

$$\frac{e}{CD(R_1 + L_1 D + 1/CD)} + \frac{1}{C^2 D^2 (R_1 + L_1 D + 1/CD)} i_2 = \left[\frac{1}{CD} + L_2 D + R_2\right] i_2$$

$$\frac{e}{CD(R_1 + L_1 D + 1/CD)} = \left[\frac{1}{CD} + L_2 D + R_2 - \frac{1}{C^2 D^2 (R_1 + L_1 D + 1/CD)}\right] i_2$$

$$\frac{i_2}{e} = \frac{1}{[CD(R_1 + L_1 D + 1/CD)]\left[\frac{1}{CD} + L_2 D + R - \frac{1}{C^2 D^2 (R_1 + L_1 D + 1/CD)}\right]}$$

$$\frac{i_2}{e} = \frac{1}{\left[(R_1+L_1D+\frac{1}{CD}) + L_2D^2C(R+L_1D+\frac{1}{CD}) + CRD(R_1+L_1D+\frac{1}{CD}) - \frac{1}{CD}\right]}$$

$$= \frac{1}{(1+L_2D^2C+CRD)(R_1+L_1D+\frac{1}{CD}) - \frac{1}{CD}}$$

Now for $\frac{i_1}{e}$,

$$① \Rightarrow e = (R+L_1D+\frac{1}{CD})i_1 - \frac{1}{CD}i_2$$

$$i_2 = \left[(R+L_1D+\frac{1}{CD})i_1 - e\right]CD$$

$$i_2 = (RCD+L_1CD^2+1)i_1 - eCD$$

$$② \Rightarrow \frac{i_1}{CD} = (L_2D+R_2+\frac{1}{CD})i_2$$

$$= (L_2D+R_2+\frac{1}{CD})(RCD+L_1CD^2+1)i_1 - eCD$$

$$eCD = (L_2D+R_2+\frac{1}{CD})(RCD+L_1CD^2+1)i_1 - \frac{i_1}{CD}$$

$$eCD = \left[(L_2CD^2+R_2CD+1)(R+L_1D+\frac{1}{CD}) - \frac{1}{CD}\right]i_1$$

$$\frac{i_1}{e} = \frac{CD}{(L_2CD^2+R_2CD+1)(R+L_1D+\frac{1}{CD}) - \frac{1}{CD}}$$

State Variables:

$e = u$

$x_1 = i_1$

$x_2 = i_2$

$x_3 = V_c$

* KVL @ loop 1:

$e = V_R + V_L + V_c$

$= i R_1 + L D i_1 + V_c$

$u = x_1 R_1 + L D x_1 + x_3$

$u = x_1 R_1 + L x_1' + x_3$

$x_1' = \frac{u}{L_1} - \frac{x_1}{L_1} R_1 - \frac{x_3}{L_1}$

* KVL @ loop 2:

$V_c = V_{L_2} + V_{R_2}$

$V_c = L_2 D i_2 + i_2 R_2$

$\Rightarrow x_3 = L_2 x_2' + x_2 R_2$

$x_2' = \frac{x_3}{L_2} - \frac{x_2}{L_2} R_2$

* KCL on node

$i_1 = i_{L_2} + i_c$

$$i_1 = i_2 + CDV_c$$

$$x_1 = x_2 + CDx_3$$

$$x_1 = x_2 + C x_3'$$

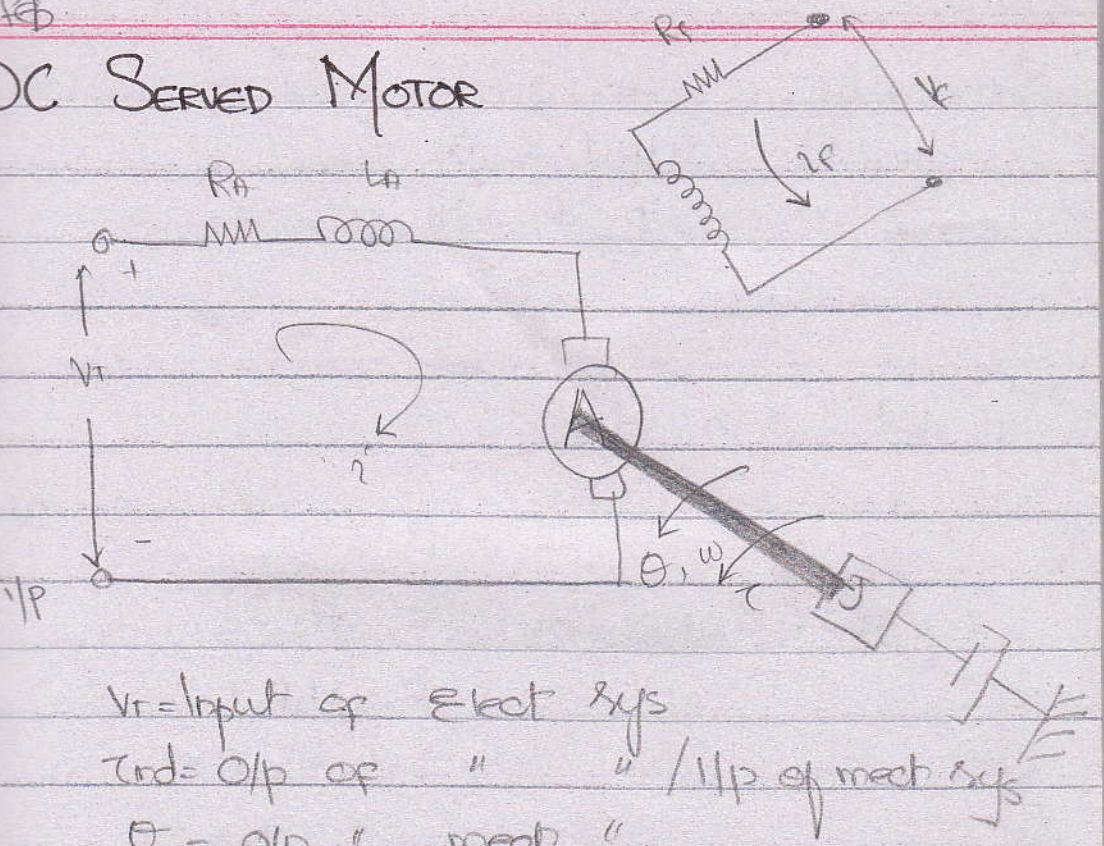
$$x_3' = \frac{x_1}{C} - \frac{x_2}{C}$$

then SMF

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} -R_1/L_1 & 0 & -1/L_1 \\ 0 & -R_2/L_2 & 1/L_2 \\ 1/C & -1/C & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1/L_1 \\ 0 \\ 0 \end{bmatrix} u$$

7th Feb

DC Served Motor



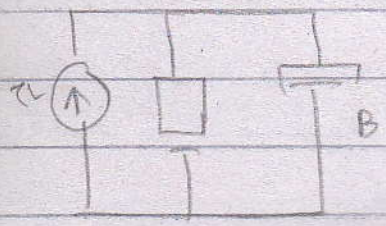
Vt = Input of Elect sys
Tind = O/p of " " / I/p of mech sys
θ = o/p " mech "

The Current Carrying Conductor or Coil in the presence of magnetic field has force (torque) induced in it. This force is directly proportional to the amount of current flowing through the Conductor, the length of the Conductor, the magnitude of magnetic flux density B and the sine of the angle blw the lengths of the Conductor & the dir of mag flux density.

* current in is responsible for generating Tind. Tind is the i/p of mechanical sys

Since

T/F is $\frac{\text{output}}{\text{i/p}} = \frac{\theta_L}{V_t}$



$T_L = JD^2\theta + BD\theta \rightarrow (1)$

$F = i(L \times B)$

$F = iLB$

$\Rightarrow T = iBr$

$= ir\frac{\phi}{A}$

$\boxed{T_{ind} = K\phi i_a} \rightarrow (2)$

* By varying values of i & ϕ we can change $T_{induced}$

(1) Armature Control Method:

$T_{ind} = K_1 i_a \rightarrow (3) \quad \because K_1 = K\phi$

Physical law ($\theta/\omega = ?$)

$T_{ind} = T_L$

$K_1 i_a = JD^2\theta + BD\theta \rightarrow (4)$

Applying

(4) $\Rightarrow K_1$

K_1

$\Rightarrow$

Applying KVL

$$V_f = i_a R_a + L_a D i_a + E_b \quad (5)$$

$$i_a = \frac{V_f - E_b}{R_a + L_a D}$$

$$(4) \Rightarrow K_1 \left(\frac{V_f - E_b}{R_a + L_a D} \right) = JD^2\theta + BD\theta$$

$$K_1 \left(\frac{V_f - E_b}{R_a + L_a D} \right) = (JD^2 + BD)\theta$$

$$\because \quad E_b = k' \phi \omega$$

$$= K_2 \omega$$

$$= K_2 D\theta$$

$$\Rightarrow K_1 \left(\frac{V_f - K_2 D\theta}{R_a + L_a D} \right) = (JD^2 + BD)\theta$$

$$\frac{K_1 V_f}{R_a + L_a D} - \frac{K_1 K_2 D\theta}{R_a + L_a D} = (JD^2 + BD)\theta$$

$$\frac{K_1 V_f}{R_a + L_a D} = \left(JD^2 + BD + \frac{K_1 K_2 D}{R_a + L_a D} \right)\theta$$

$$\frac{\theta}{V_f} = \frac{K_1}{(R_a + L_a D)\left[JD^2 + BD + \frac{K_1 K_2 D}{R_a + L_a D} \right]}$$

$$= \frac{K_1}{JD^2(R_a + L_a D) + BD(R_a + L_a D) + K_1 K_2 D}$$

$$= \frac{K_1}{JD^2R_A + JL_AD^3 + BDR_A + BD^2L_A + K_1K_2D}$$

$$= \frac{K_1}{D^3JL_A + D^2(JR_A + BL_A)D^2 + (BR_A + K_1K_2)D}$$

$$\frac{\theta}{V_1} = \frac{K_1}{JL_AD^3 + (JR_A + L_AB)D^2 + (BR_A + K_1K_2)D}$$

S.V

$u = V_1$

$x_1 = i_A$

$x_2 = \theta$

$x_3 = \omega \quad = D\theta = Dx_2 = x_2' = x_3$

* For Current Consider KVL eq

⑤ $\Rightarrow u = x_1R_A + L_ADx_1 + K_2\omega$

$u = x_1R_A + L_Ax_1' + K_2x_3$

$x_1' = \frac{u}{L_A} - \frac{x_1R_A}{L_A} - \frac{K_2x_3}{L_A}$

(4) $\Rightarrow k_1 i_A = J D\omega + B\omega$

$\Rightarrow k_1 X_1 = J X_3' + B X_3$

$X_3' = \frac{k_1}{J} X_1 - \frac{B}{J} X_3$

load is ↑ speed

$\tau_{load} > \tau_{ind}$

$\omega \downarrow$

$V_t \uparrow$

$i_A \uparrow = \frac{V_t - E_A}{R_A + L_A D}$

$\tau_{ind} \uparrow = k_1 i_A \uparrow$

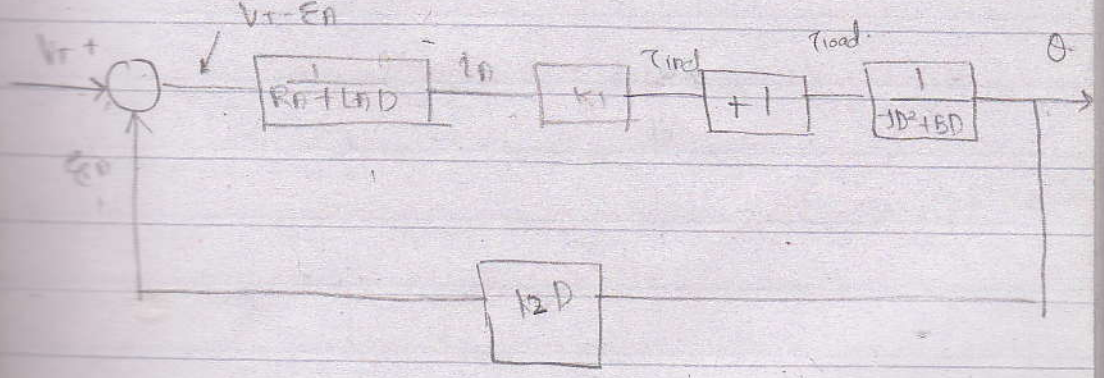
$\tau_{ind} > \tau_{load}$

$\omega \uparrow$

$E_A \uparrow = k\Phi\omega \uparrow$

$i_A \downarrow$

$\Rightarrow \tau_{ind} = \tau_{load}$



(ii) FIELD CONTROL METHOD. θ / V_f

$$\tau_{ind} = k \Phi i_a$$

* Const but Φ can't be the bus

$$\tau_{ind} = k i_a \, k' i_f$$

$$\tau_{ind} = k_3 \, i_f$$

Physical law

$$\tau_{ind} = \tau_L$$

$$k_3 i_f = JD^2\theta + BD\theta \quad \rightarrow (5)$$

KVL

$$V_f = R_f i_f + L_f D i_f$$

$$V_f = i_f (R_f + L_f D)$$

$$i_f = \frac{V_f}{R_f + L_f D}$$

$$(5) \Rightarrow k_3 \left(\frac{V_f}{R_f + L_f D} \right) = JD^2\theta + BD\theta$$

$$\frac{k_3 V_f}{R_f + L_f D} = \theta (JD^2 + BD)$$

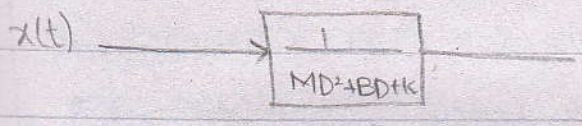
$$\frac{\theta}{V_f} = \frac{k_3}{(R_f + L_f D)(JD^2 + BD)}$$

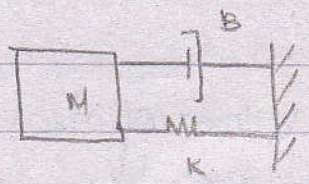
10th feb 2014

CHAPTER 3:

SOLUTION OF DIFFERENTIAL EQUATIONS

Roots (poles) will depire the nature of respons



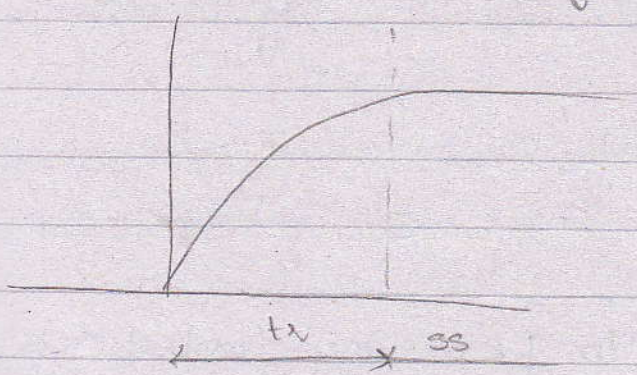


$f = (MD^2 + BD + K)x$

$$c(t) = \underset{PS}{c(t)} + \underset{C.F}{c(t)} = \underset{ss}{c(t)} + \underset{tr}{c(t)}$$

PS → particular sol

CF → Complementary fun



* Dynamic eq depends upon i/p but not on i/p type

- Steady state funct / component has the same form as that of driving funct

  ie if i/p is sinusoid then o/p will be sinusoid & same for other form of diff shape i/p

- The instantaneous value of transient component depends on boundary cond the roots of characteristic eq & the instantaneous value of steady state comp.

  boundary cond → sys @ rest

  ie

  $x = 0$

  $v = 0$

- The no of i/p from diff control sys can be computed from the following standard inputs

Types Of I/p :-

Sinusoidal i/p $r = \cos\omega_0 t$

Power Series funct $r = a_0 + a_1 t + \frac{a_2}{2} t^2$

Step i/p " $r = u(t)$

Ramp " " $r = t\,u(t)$

parabolic " " $r = t^2\,u(t)$

Impulse " " $r = \delta(t)$

Exponential i/p

Why singularity

Cor if One funct is differentiated Other is acheived

* In time domain

e^{+ve} Unstable

e^{-ve} Stable

power of exp is used to judge whether sys is stable or not

14th Feb 2014

Steady State Response

Consider a Integro-differential eq.

$$[A_v D^v + \cdots + A_0 + \cdots + A_{-w} D^{-w}]\ c(t) = r(t) \quad \text{①}$$

For polynomial input (power series)
Where

$$\text{input} \rightarrow r(t) = r_0 + r_1 t + \frac{r_2 t^2}{2!} + \cdots + \frac{r_k t^k}{k!} \quad \text{②}$$

$$\text{output} \rightarrow c(t) = b_0 + b_1 t + \frac{b_2 t^2}{2!} + \cdots + \frac{b_q t^q}{q!} \quad \text{③}$$

$$q = k + x$$

Where

q = highest power appearing in o/p

x = lowest " " " int. diff. eq.

For eg a mech sys is

$$(MD^2 + BD + K)\, x(t)_{ss} = f$$

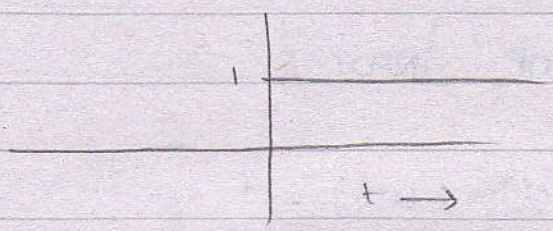
Generally

$$(A_2 D^2 + A_1 D + A_0)\, c(t)_{ss} = r(t) \rightarrow \text{④}$$

* Steady state sure has the form of forcing function

* both i/p & o/p ie r(t) & c(t) r of same shape but mag values.

STEP INPUT



$r(t) = u(t) = t^0 u(t) = 1$ * ⇒ k = 0

* mostly i/p r step.

then

$q = k + x$

$q = 0 + 0$

$x = 0$

∵ lowest power of D in eq 4 is zero

from (3)

$c(t)_{ss} = b_0$

Subs in (4)

$(a_2D^2 + a_1D + a_0)b_0 = 1$

$0 + 0 + A_0 b_0 = 1$

$b_0 = \frac{1}{A_0}$

$$\Rightarrow \boxed{c(t)_{ss} = b_0 = \frac{1}{A_0}}$$

OR

$$\boxed{c(t)_{ss} = b_0 = \frac{1}{k}}$$

RAMP INPUT

$r(t) = t\,u(t) = 1$

here

$k = 1$

$q = k + x$

$q = 1 + 0 = 1$

from (3)

$c(t) = b_0 + b_1 t$

Subs in (4)

$(A_2 D^2 + A_1 D + A_0)(b_0 + b_1 t) = 1$

$(0 + 0 + A_0 b_0)\ A_2 b_1 (0) + A_1 b_1 + A_0 b_1) = 1$

$A_1 b_1 + A_0 b_0 + A_0 b_1 t = 1$

A Comparing Coeff

$$A_1 b_1 + A_0 b_0 = 0 \quad , \qquad A_0 b_0 = 1$$

$$A_1\left(\frac{1}{A_0}\right) + A_0 b_0 = 0 \qquad b_1 = \frac{1}{A_0}$$

$$\frac{A_1}{A_0} + A_0 b_0 = 0$$

$$b_0 = -\frac{A_1}{A_0^2}$$

$$\Rightarrow c(t)_{ss} = -\frac{A_1}{A_0^2} + \frac{1}{A_0} t$$

$$c(t)_{ss} = -\frac{B}{K^2} + \frac{1}{K} t$$

PARABOLIC INPUT :

$$r(t) = \frac{t^2}{2!} u(t) = \frac{t^2}{2} u(t)$$

here

$$K = 2$$

then

$$q = 2 + 0$$

$$q = 2$$

from ②

$$c(t) = b_0 + b_1 t + \frac{b_2 t^2}{2}$$

Subs in eq 4

$$(A_2D^2 + A_1D + A_0)\left(b_0 + b_1t + \frac{b_2t^2}{2}\right) = \frac{t^2}{2}$$

$$A_2b_2 + A_1b_1 + A_1b_2t + A_0b_0 + A_0b_1t + \frac{A_0b_2t^2}{2} = \frac{t^2}{2}$$

$$\frac{A_0b_2t^2}{2} + t(A_1b_2 + A_0b_1) + (A_2b_2 + A_1b_1 + A_0b_0) = \frac{t^2}{2}$$

Comparing Coeff

$$\Rightarrow \frac{A_0b_2}{2} = \frac{1}{2} \qquad \Rightarrow A_0b_1 + A_1b_2 = 0$$

$$\Rightarrow \boxed{b_2 = \frac{1}{A_0}} \qquad A_0b_1 + A_1\left(\frac{1}{A_0}\right) = 0$$

$$A_0b_1 = -\frac{A_1}{A_0}$$

$$\boxed{b_1 = -\frac{A_1}{A_0^2}}$$

$$\Rightarrow A_2b_2 + A_1b_1 + A_0b_0 = 0$$

$$\frac{A_2}{A_0} - \frac{A_1^2}{A_0} = -A_0b_0$$

CASE I:

→ When

Sys Re

and the

$$A_0 b_3 = \frac{A_1^2}{A_0} - \frac{A_2}{A_0}$$

$$\boxed{b_0 = \frac{A_1^2}{A_0^2} - \frac{A_2}{A_0^2}}$$

Transient Response

$$[A_n D^n + \cdots A_0 + \cdots A_{-\omega} D^{-\omega}]\, c(t) = r(t).$$

$$r(t) = 0 \qquad D \to m$$

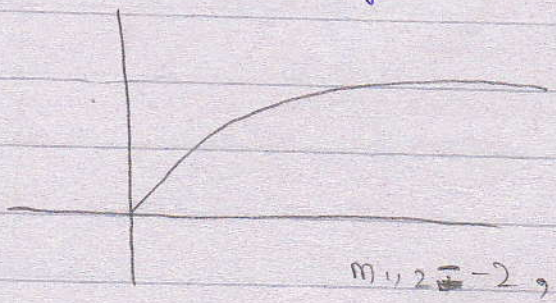
$$[A_n m^n + \cdots + A_0 + \cdots A_{-\omega} m^{-\omega}]\, c(t)_{tr} = 0$$

$$c(t)_{tr} = Ae^{-mt}$$

$$Mm^2 + Bm + k = 0.$$

Case I:

→ When the Roots are real, neg & distinct Sys Response: Over damped and the sys response is very slow.



$m_{1,2} = -2, -3$

$$c(t)_{tr} = A_1 e^{-2t} + A_2 e^{-3t}$$

Case II:

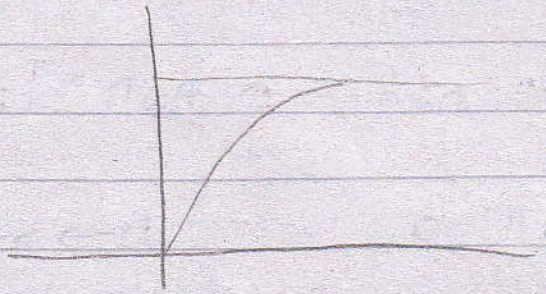
When the sys roots are real, neg and equal

Sys Response: Critically damped.

and faster than over damped

$$m_{1,2} = -2, -2$$

$$C(t)_{tr} = A_1 e^{-2t} + A_2 e^{-2t}$$



Case III

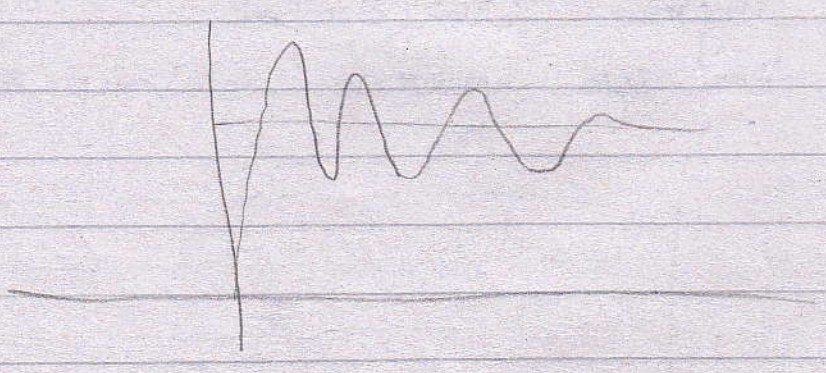
When the roots r Complex & the real part is negative

Sys Response: Under damped

& it is faster than Critically damped.

$$m_{1,2} = -\theta \pm j\omega$$

$$C(t)_{tr} = A_1 e^{(\theta + j\omega)t} + A_2 e^{(\theta - j\omega)t}$$



$$= A_1 e^{-\theta t} e^{j\psi t} + A_2 e^{-\theta t} e^{-j\omega t}$$

Since

$$A_1 = A_0 \angle \psi \quad = A_0 e^{j\psi}$$

$$A_2 = A_0 \angle -\psi \quad = A_0 e^{-j\psi}$$

then

$$A_0 e^{-\theta t} e^{j\psi} \cdot e^{j\omega t} + A_0 e^{-j\psi} e^{-j\omega t} e^{-\theta t}$$

$$= 2A_0 e^{-\theta t} \left[e^{j\psi} \cdot e^{j\omega t} + e^{-j\psi} \cdot e^{-j\omega t} \right]$$

Now

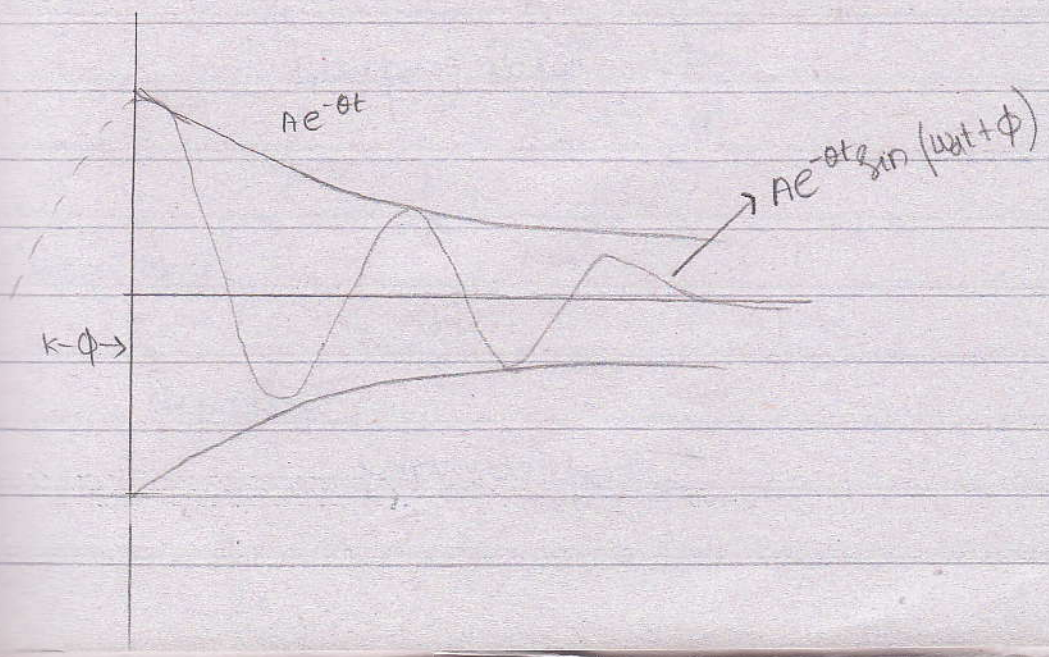
$$c(t)_{tr} = A e^{-\theta t} \cos(\omega t + \psi)$$

$$= A e^{-\theta t} \sin(\omega t + \psi + 90^\circ)$$

$$= A e^{-\theta t} \sin(\omega t + \phi)$$

where

$$\phi = \psi + 90^\circ$$



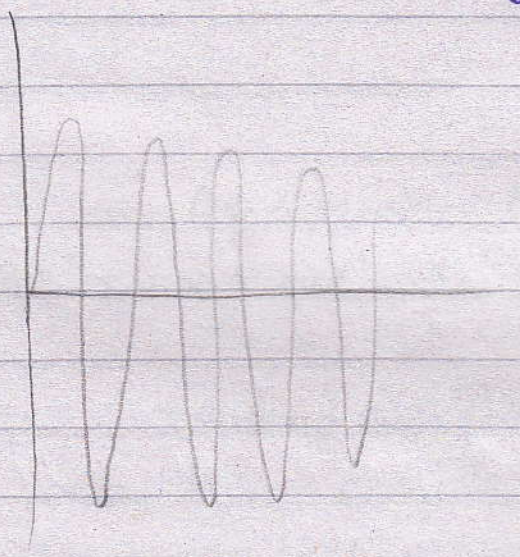
CASE IV:

When the roots r imaginary & sys real part is zero.

Sys Response: Undamped. Sinusoid and oscillatory with sys natural freq

$$c(t)_{tr} = A \sin(\omega_d t + \phi) e^{-\theta t}$$

The sys is called marginally stable sys.



17th feb

Damp

N

Consider

Apply

Comple

Where

CASE # 1

Sys

17th Feb

Damping Ratio Zeta (ζ) and Undamped Natural Frequency (ω_n)

Consider a sys

$$b_2 m^2 + b_1 m + b_0 = 0$$

↳ 4x transient resp i/p is zero

Apply Quadratic formula

$$m_{1,2} = \frac{-b_1 \pm \sqrt{b_1^2 - 4b_0 b_2}}{2b_2}$$

Complex Roots

$$m_{1,2} = \frac{-b_1 \pm j\sqrt{b_1^2 - 4b_0 b_2}}{2b_2}$$

Where

b_1 = Actual value of damping

$2\sqrt{b_0 b_2} = b_1'$ = Critical value of 4

$$\text{Damping Ratio} = \frac{b_1}{b_1'}$$

CASE # 1:

$b_1 > b_1'$ $\quad \zeta > 1$

Sys Resp:

Over damped

Case #2

$b_1 = b_1' \quad \Rightarrow \zeta = 1$

Sys Response:

Critically damped

Case #3

$b_1 < b_1' \quad \zeta < 1$

Sys Response

Under damped

Case #4

$b_1 = 0 \quad \Rightarrow \zeta = 0$

Sys Resp

Under damped with sustainable oscillations

By ↑ing value of b_1 damping ↑ses ∴ hence system response is slower ∴ sluggish

$$m_{1,2} = \frac{0 \pm \sqrt{0 - 4 b_0 b_2}}{2 b_2}$$

$$m_{1,2} = \pm j \sqrt{\frac{b_0}{b_2}}$$

$$\omega_n = \sqrt{b_0/b_2} \quad (*)$$

* Under damped resp is fast, but in this case we have to face overshoots

$$b_2 m^2 + b_1 m + b_0 = 0$$

Dividing by b_2

$$m_1^2 + \frac{b_1}{b_2} m + \frac{b_0}{b_2} = 0$$

$$m_1^2 + \frac{2b_1}{2b_2} \times \frac{\sqrt{b_0}}{\sqrt{b_2}} + \frac{b_0}{b_2} = 0$$

$$m^2 + 2\left(\frac{b_1}{2\sqrt{b_0 b_2}}\right)\left(\sqrt{\frac{b_0}{b_2}}\right) + \frac{b_0}{b_2} = 0$$

$$\Rightarrow \boxed{m^2 + 2\zeta\omega_n m + \omega_n^2 = 0}$$ system control eq

Roots are:

$$m_{1,2} = \frac{-\zeta\omega_n \pm \sqrt{(2\zeta\omega_n)^2 - 4(1)(\omega_n)}}{2(1)}$$

$$m_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

* When b_1 comes near to zero, sustained oscillations is achieved.

21st feb 2014

Consider

$$m^2 + 6m + 2 = 0$$

Find

ξ and ω_n

here

$2\xi\omega_n = 6$ and $\omega_n^2 = 2$

$\omega_n = \sqrt{2}$

$$\xi = \frac{6}{2\omega_n}$$

$$\xi = \frac{6}{2\sqrt{2}}$$

$$\xi = \frac{3}{\sqrt{2}}$$

$\Rightarrow$ Sys Resp is Under damped

$$m_{1,2} = -3 \pm j\sqrt{2}\sqrt{1 - 3/2}$$

$$= -3 \pm \sqrt{2}\, j\left(\sqrt{-1/2}\right)$$

$$= -3 \pm \frac{\sqrt{2}}{\sqrt{2}}\, j^2$$

$$= -3 \pm (-1)$$

$m_1 = -3 - 1$

$m_1 = -4$

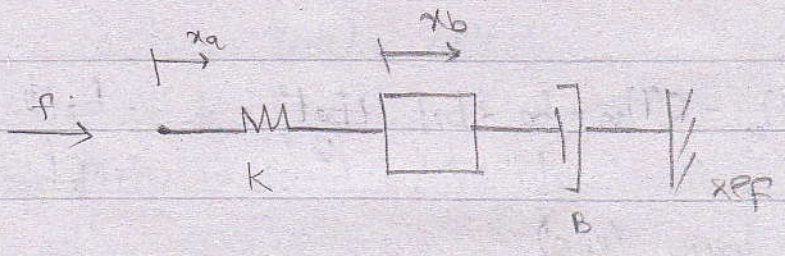
$m_2 = -3 + (-1)$

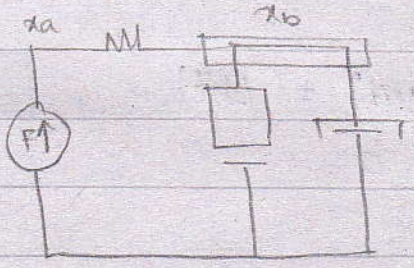
$= -2$

21st feb 2014

2ND ORDER MECHANICAL SYSTEM

Consider a system (2nd Order)





@ x_a $\quad F = k(x_a - x_b)$

@ x_b $\quad K(x_a - x_b) = MD^2x_b + BDx_b$

$$kx_a = (MD^2 + BD + k)x_b \quad \rightarrow (1)$$

Response (total)

$$x_b(t)_{tot} = x_b(t)_{s.s} + x_b(t)_{tr} \quad \rightarrow (2)$$

STEADY STATE RESPONSE:

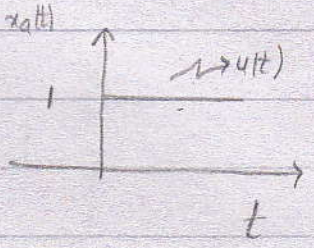
$$x_a(t) = r(t) = r_0(t) + r_1(t) + \frac{r_2 t^2}{2!} + \cdots \frac{r_k t^k}{k!} \quad (3)$$

$$\Rightarrow x_b(t)_{ss} = c(t)_{ss} = b_0 + b_1 t + \frac{b_2 t^2}{2!} + \cdots \frac{b_q t^q}{q!} \quad (4)$$

STEP INPUT ($k=0$)

3) $\Rightarrow$ $x_a(t) = u(t) = t^0 u(t)$

$\Rightarrow k = 0$



4) $\Rightarrow$ $x_b(t)_{ss} = b_0$

Subs in dynamic eq (i)

1) $\Rightarrow k(1) = M\cancel{D^2}^{0} b_0 + B\cancel{D}^{0} b_0 + K b_0$

$k(1) = k(b_0)$

$1 = b_0$

(4) $\Rightarrow$ $\boxed{x_b(t)_{ss} = b_0 = 1}$

Transient Response:

Putting $x_a(t) = 0$

$D \longrightarrow m$, then $x_b(t) = x_b(t) e^{rt}$

eq ① $\Rightarrow (Mm^2 + Bm + K) x_b(t) e^{rt} = 0$

$\Rightarrow Mm^2 + Bm + K = 0$

-ing throughout by M

$$m^2 + \frac{B}{M} m + \frac{K}{M} = 0$$

Comparing with

$$m^2 + 2\xi\omega_n m + \omega_n^2 = 0$$

$$\Rightarrow \omega_n = \sqrt{\frac{K}{m}}$$

then

$$\xi = \frac{B}{2M}\sqrt{M} = \frac{B}{2\sqrt{KM}}$$

CASE I:

Over damped

$\xi > 1$

$B > 2\sqrt{MK}$

$$x_b(t)_{IN} = A_1 e^{m_1 t} + A_2 e^{m_2 t} \quad (A)$$

$$m_{1,2} = \frac{-2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2 - 4(1)\omega_n^2}}{2}$$

$$= \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= \frac{-2\xi\omega_n \pm 2\omega_n\sqrt{\xi^2 - 1}}{2}$$

$$= \omega_n\left(-\xi \pm \sqrt{\xi^2 - 1}\right)$$

$$\Rightarrow x_b(t)_{IR} = A_1 e^{(-\xi+\sqrt{\xi^2-1})\omega_n t} + A_2 e^{(-\xi-\sqrt{\xi^2-1})\omega_n t}$$

CASE II :-

-then

(A) $\Rightarrow x_b(t)$

CASE III:

$\rightarrow$ Ques

needs

$\Rightarrow x_b(t)_{IR}$

CASE II :-

Critically damped

$\xi = 1$

$B = 2\sqrt{MK}$

then

$m_{1,2} = -\omega_n$

(A) $\Rightarrow x_{(t)_{tr}} = A_1 e^{-\omega_n t} + A_2 e^{-\omega_n t}$

$= A_1 e^{-\sqrt{\frac{K}{M}} t} + A_2 e^{-\sqrt{\frac{K}{M}} t}$

CASE III:

Under damped (most demanding)

→ Overshoot, rise time, peak times all needs to be considered.

$\xi < 1$

$B < 2\sqrt{MK}$

then

$m_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

$= -\left(\xi \pm j\sqrt{1-\xi^2}\right)\omega_n$

$\Rightarrow x_{(t)_{tr}} = Ae^{-\sigma t} \sin(\omega_d t + \phi)$

$= Ae^{-\xi\omega_n t} \sin(\omega_n\sqrt{1-\xi^2}\, t + \phi)$

then

$x_b(t)_{total}$ = 1 + ...

↓ transient Response

CASE IV

→ Roots imaginary

→ Real part = 0

$\xi = 0$

we get

$$m_{1,2} = \pm j\omega n$$

then

$$x_b(t) = A \sin(\omega n t + \phi)$$

FOR UNDERDAMPED RESPONSE:

$$x_b(t)_{tot} = 1 + Ae^{-\xi\omega_n t} \sin(\omega_n\sqrt{1-\xi^2}\,t + \phi)$$

Now:

We have to find A and ϕ.

@ $t = 0$,

$$x_b(t) = 0$$

$$\dot{x}_b(t) = 0$$

putting cond(i) in above eq

$$x_b(t) = 1 + A\sin(\phi) = 0$$

$$\boxed{A = \frac{-1}{\sin\phi}} \quad (6)$$

Differentiating eq

$$Dx_b(t) = A\left[e^{-\xi\omega_n t}\cos\left(\omega_n\sqrt{1-\xi^2}\,t + \phi\right)\left\{\omega_n\sqrt{1-\xi^2}\right\} + \sin\left(\omega_n\sqrt{1-\xi^2}\,t + \phi\right)e^{-\xi\omega_n t}\left(-\xi\omega_n\right)\right]$$

@ $t=0$ $\quad Dx_b(t) = 0$

$$\Rightarrow A\left[(1)\cos\phi\left\{\omega_n\sqrt{1-\xi^2}\right\} + \sin\phi(+1)\left(-\xi\omega_n\right)\right] = 0$$

$$\cos\phi\,\omega_n\sqrt{1-\xi^2} = \sin\phi\,\xi\omega_n$$

$$\cancel{\omega_n}\sqrt{1-\xi^2}\cos\phi = \sin\phi\,\xi\cancel{\omega_n}$$

$$\frac{\sin\phi}{\cos\phi} = \frac{\sqrt{1-\xi^2}}{\xi}$$

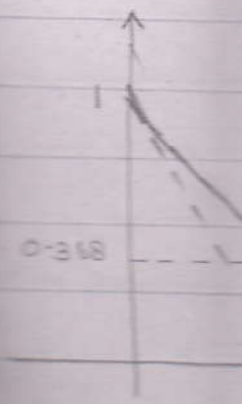
$$\tan\phi = \frac{\sqrt{1-\xi^2}}{\xi} \qquad A = \frac{-1}{\sqrt{1-\xi^2}}$$

Puting in eq

$$\boxed{x_b(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left(\omega_n\sqrt{1-\xi^2} + \cos^{-1}\xi\right)}$$

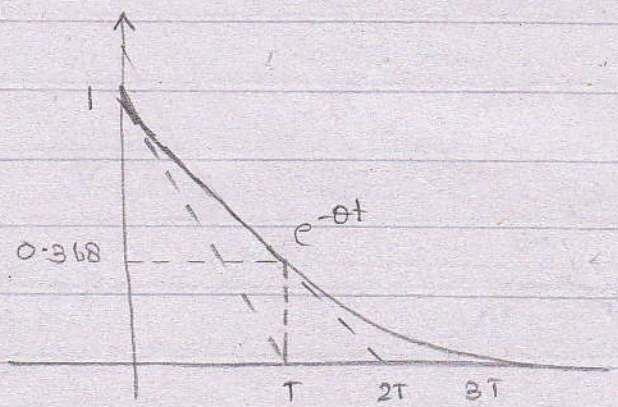
$A = \frac{1}{\sin\phi} = \frac{\xi\cos\phi}{\sqrt{1-\xi^2}}$

Time Constan



The time c
'e' becom
as time c
In this ti
it's final

Time Constant



The time at which the power of exp 'e' becomes equal to -1 is known as time const.
In this time the systems energy from it's final value is 36.8%.

$$-\theta t = -1$$

@ this time

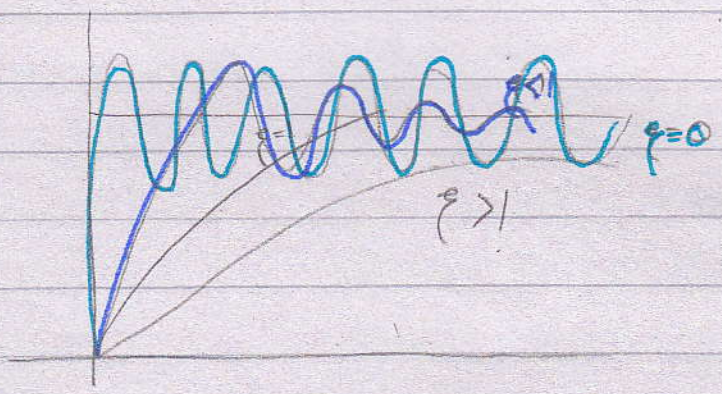
$$t = T$$

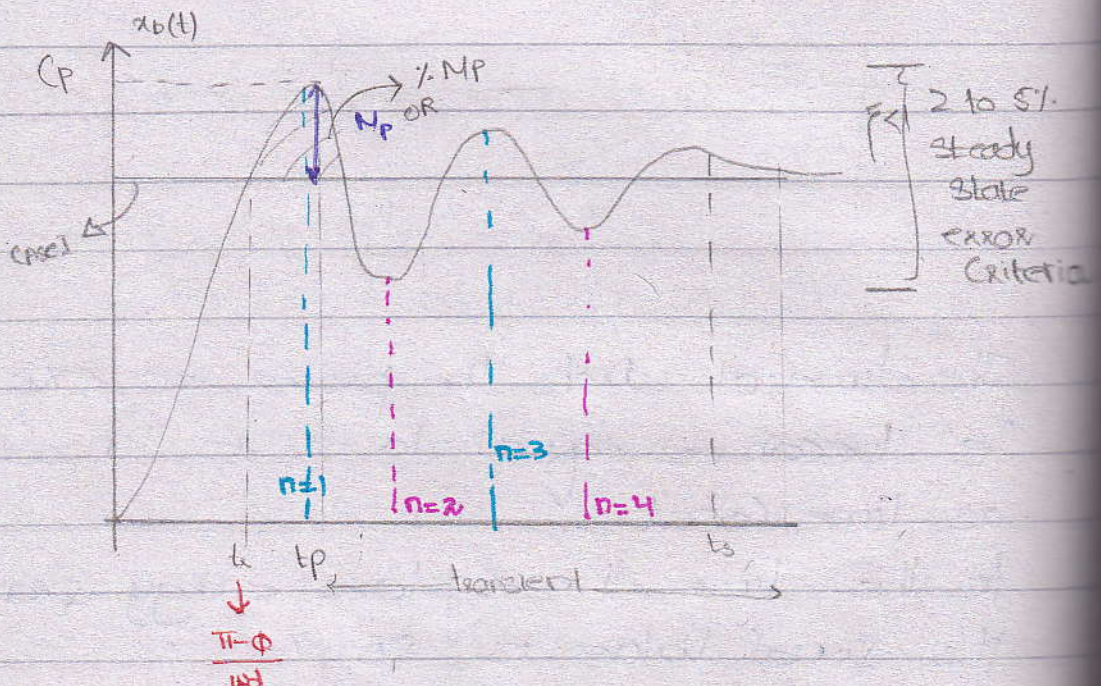
what

$$T = \frac{1}{\theta} = \frac{1}{\zeta\omega_n}$$

Remaining Energy = 36.8
lost " = 63.2

28TH Feb





t_p = Peak time

M_p = Rep overshoot

t_r = rise time

t_s = Settling time

C_p = Peak magnitude

Rise Time

Rise time is a time at wch system reaches 2 it's value for the first time

When

$$x_b(t) = 1$$
$$t = t_r$$

then

$$x_b(t) = 1 - \frac{e^{-\xi\omega_n}}{\sqrt{1-\xi^2}} \sin\left(\omega_n\sqrt{1-\xi^2}\,t + \cos^{-1}\xi\right)$$

$$1 = 1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \phi)$$

$$\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \phi) = 0$$

$$\Rightarrow e^{-\xi\omega_n t_r} = 0$$

$$-\xi\omega_n t_r = 0$$

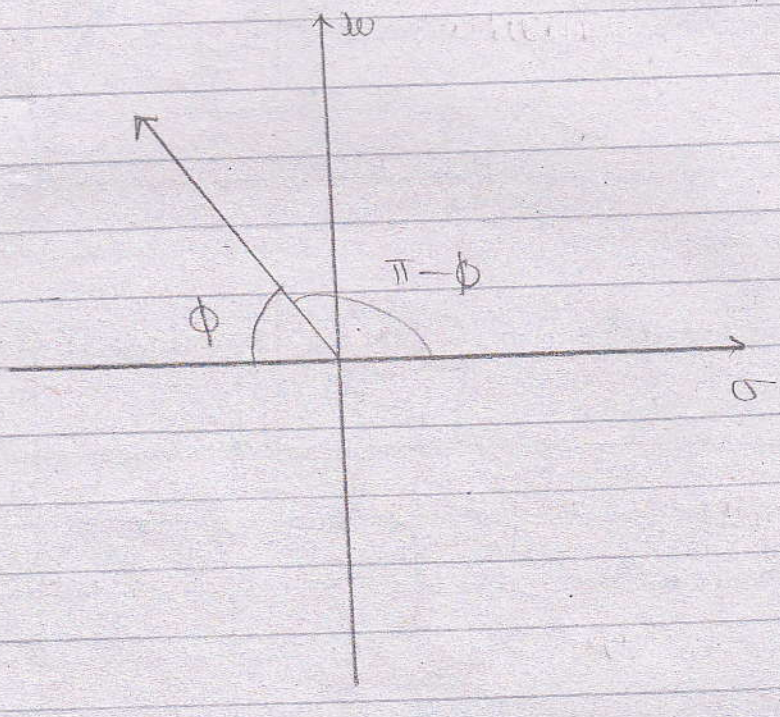
$$t_r = \frac{0}{-\xi\omega_n}$$

$$\Rightarrow \sin(\omega_d t_r + \phi) = 0$$

$$\sin\omega_d t_r \cos\phi = -\cos\omega_d t_r \sin\phi$$

$$\frac{\sin\omega_d t_r}{\cos\omega_d t_r} = \frac{\sin\phi}{\cos\phi} = -\frac{\sqrt{1-\xi^2}}{\xi}$$

$$t_s = \frac{\pi - \phi}{\omega_d} = \frac{\pi - \omega_n \zeta}{\omega_n \sqrt{1-\zeta^2}}$$



2ND METHOD:

$$e^{-\xi \omega_n t} \sin(\omega_d t_r + \phi) = 0$$

$$\Rightarrow \sin(\omega_d t_r + \phi) = 0$$

$$\omega_d t_r + \phi = \sin^{-1} 0$$

$$\omega_d t_r + \phi = 0, n\pi$$

$$\Rightarrow \omega_d t_r = 0 - \phi$$

$$t_r = \frac{-\phi}{\omega_d}$$ ✗ ∵ t_r can't be -ve

$$\Rightarrow \omega_d t_r = n\pi - \phi$$

$$t_r = \frac{1\pi - \phi}{\omega_d}$$

Peak Time:

* Paper fig 2 of lecture on 28th Feb.

$$x_b(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left(\omega_n\sqrt{1-\zeta^2}\,t + \cos^{-1}\zeta\right)$$

peak time:
time @ wch overshoot arrives

* For finding peak time
peak time is the time at wch first or max overshoot comes. It can be find out by differentiating $x_b(t)$ and setting it equals to zero.

$$t_p = \frac{\pi}{\omega_d}$$

here
let

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\phi = \cos^{-1}\zeta$$

then

$$x_b(t) = 1 - e^{-\zeta\omega_n t}\left[\sin\omega_d t\cos\phi + \cos\omega_d t\sin\phi\right]$$

$\sin\phi$ $\sin\phi$

$$x_b(t) = 1 - e^{-\xi\omega_n t}\left[\frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t + \cos\omega_d t\right]$$

U V

Now

Differentiating w.r.t "t"

$$\frac{dx_b(t)}{dt} = 0 = 0 - \left[e^{-\xi\omega_n t}\left\{\frac{\xi\omega_d}{\sqrt{1-\xi^2}}\cos\omega_d t - \sin\omega_d t(\omega_d)\right\} + \left(\frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t + \cos\omega_d t\right)e^{-\xi\omega_n t}(-\xi\omega_n)\right]$$

$$= \quad \because \omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= -e^{-\xi\omega_n t}\left[\xi\omega_n\cos\omega_d t - \sin\omega_d t\,\omega_d - \frac{\xi^2}{\sqrt{1-\xi^2}}\omega_n\sin\omega_d t - \xi\omega_n\cos\omega_d t\right]$$

$$= e^{-\xi\omega_n t}\left[-\sin\omega_d t\,\omega_d - \frac{\xi^2}{\sqrt{1-\xi^2}}\omega_n\sin\omega_d t\right]$$

$$0 = -\omega_n\sin\omega_d t\, e^{-\xi\omega_n t}\left[\sqrt{1-\xi^2} + \frac{\xi^2}{\sqrt{1-\xi^2}}\right]$$

$\Rightarrow \sin \omega_d t_p = 0$

$\omega_d t_p = \sin^{-1}(0)$

$\omega_d t_p = 0, n\pi$

$t_p = 0 \qquad t_p = \dfrac{n\pi}{\omega_d}$

* All overshoot occurs at odd values of n.
* Peak overshoot occurs at n=1
* All Undershoots " " even " of n.

put $t_p = \dfrac{\pi}{\omega_d}$ in ⑤ to get C_p

$$C_p = 1 - \frac{e^{-\xi\omega_n\pi/\omega_n\sqrt{1-\xi^2}}}{\sin\phi} \sin\left(\omega_n\sqrt{1-\xi^2}\,\frac{\pi}{\omega_n\sqrt{1-\xi^2}} + \phi\right)$$

$$= 1 - \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sin\phi} \sin(\pi - \sin\phi)$$

$\pi - \sin\phi$

$$= 1 + e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$C_p = 1 + \exp\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)$$

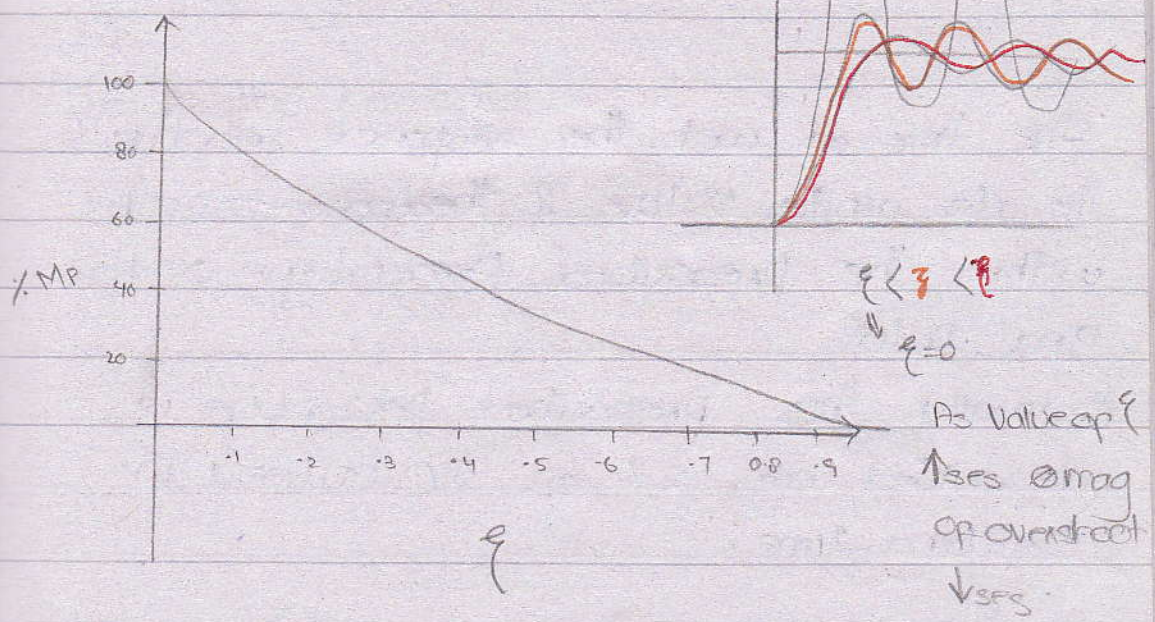
* C_p depends only on ξ

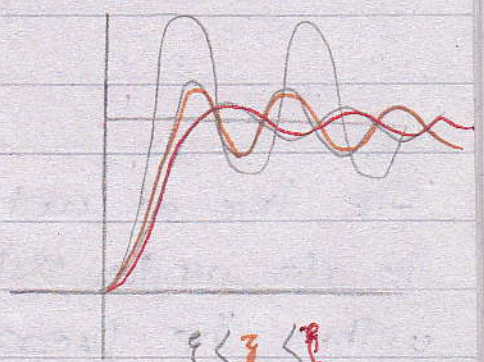
% of overshoot $\rightarrow$ $\%M_p = \frac{C_p - C_{ss}}{C_{ss}} \times 100$

$$= \frac{1 + e^{-\xi\pi/\sqrt{1-\xi^2}} - 1}{1} \times 100$$

$$= \left(e^{-\xi\pi/\sqrt{1-\xi^2}}\right) \times 100$$

$$\%M_p = 100\, e^{-\xi\pi/\sqrt{1-\xi^2}}$$





$\xi < 1 < \xi$

$\downarrow$

$\xi = 0$

As value of ξ ↑ses ⊗mag of overshoot ↓ses

* CRITICALLY DAMPED

↳ $\xi = 0$

⇓ Max M_p is achieved at $\xi = 0$

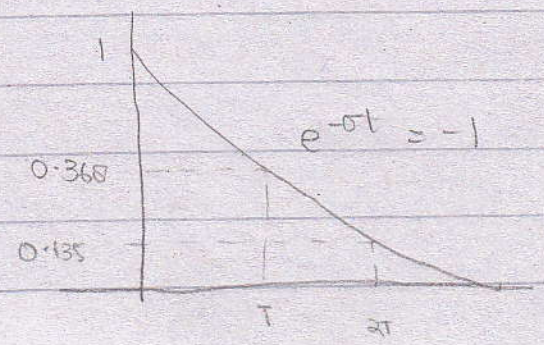
Setting Time: (t_s)

Exponential Values:

Time	$e^{-\sigma t} = e^{-\zeta\omega_n t}$	Error
1T	0.368	36.8 %
2T	0.135	13.5 %
3T	0.05	5 %
4T	0.018	1.8 %
5T	0.007	0.7 %

The time at which the response reaches to its finite value & therefore remains within the prescribed percentage of the final value.

Normally this prescribed percentage is described as 2% or 5% criteria for settling time.



$t = T$

$\sigma T = 1$

$e^{-\sigma t} = -1$

$T = \frac{1}{\sigma}$

$$t_s \text{ For 5\%} = 3T = \frac{3}{\sigma} = \frac{3}{\zeta \omega_n}$$

$$t_s \text{ For 2\%} = 4T = \frac{4}{\sigma} = \frac{4}{\zeta \omega_n}$$

Consider a system where $\zeta = 0.5$ and $\omega_n = 8.5$ rad/sec obtain.

- t_r
- t_p
- C_p
- M_p
- $t_{s\,2\%}$
- $t_{s\,5\%}$

plot the response

$$t_r = \frac{\pi - \phi}{\omega_d}$$

Where

$$\omega_d = \omega_n \sqrt{1-\zeta^2} = 4.33$$

$$\phi = \cos^{-1}\zeta = 60^\circ = \frac{\pi}{3}$$

then

$$t_r = \frac{\pi - \pi/3}{4.33} = 0.483 \text{ sec}$$

and

$$t_p = \frac{\pi}{\omega_d}$$

$$= \frac{\pi}{4.33} = 0.7255 \text{ sec}$$

then

$$C_p = 1 + \exp\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)$$

$$= 1 + e^{-1.814}$$

$$C_p = 1.163$$

Also

$$\%.M_p = \frac{C_p - C_{ss}}{C_{ss}} \times 100$$

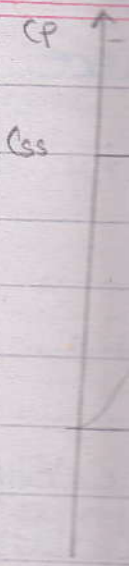
$$= 100\, e^{-\xi\pi/\sqrt{1-\xi^2}}$$

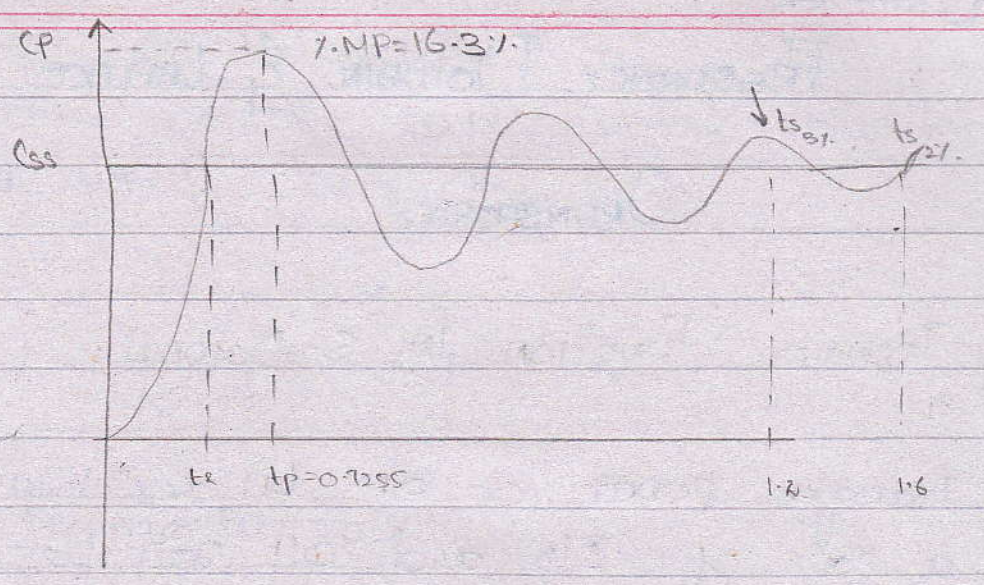
$$= 16.3\ \%$$

Now

$$t_{s_{2\%}} = 4T = \frac{4}{\xi\omega_n} = 1.6 \text{ sec}$$

$$t_{s_{5\%}} = 3T = \frac{3}{\xi\omega_n} = 1.2 \text{ sec}$$





* Steady State resp of Sinusoidal i/p.

Eg # 3.7/3.6

CHAPTER 4:

10th march

FREQUENCY DOMAIN & LAPLACE TRANSFORM

* TRANSFER FUNCTION IN S-DOMAIN

Transfer funct of an LTI sys b/w a pair of I/p and o/p variables is the ratio of the laplace transf of the o/p to the I/p for all all initial conditions are set to be zero.

let us consider a system

$$D^3 x(t) + 4D^2 x(t) + 6D x(t) + 4x(t) = (D+6) f(t)$$

$$\frac{x(t)}{f(t)} = \frac{D+6}{D^3 + 4D^2 + 6D + 4}$$

In freq dom if initial cond ~ zero

$$\frac{X(s)}{F(s)} = \frac{S+6}{S^3 + 4S^2 + 6S + 4}$$

- ZERO's OF A FUNCTION
  - ↳ zero's are the points where value of a funct becomes zero

$$s+6=0$$
$$\boxed{s=-6}$$

  - ↳ Represented by o

- Poles OF A FUNCTION
  - ↳ points where value of a function becomes infinite are poles of a function

$$s^3+4s^2+6s+4=0$$

$$\boxed{s=-2,\ -1\pm j}$$

  - ↳ Represented by x

CHARACTERSTIC EQUATION:

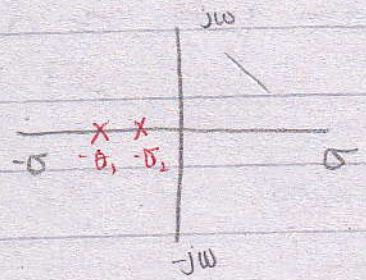
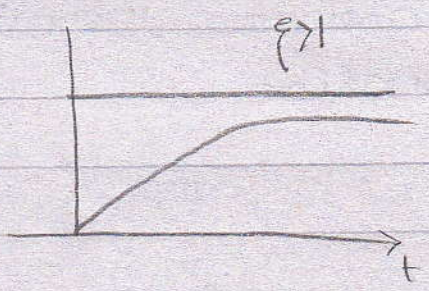
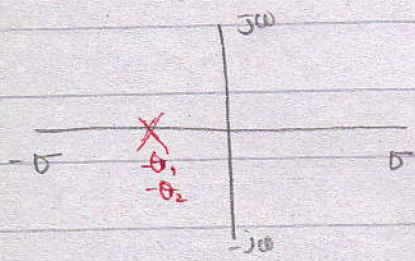
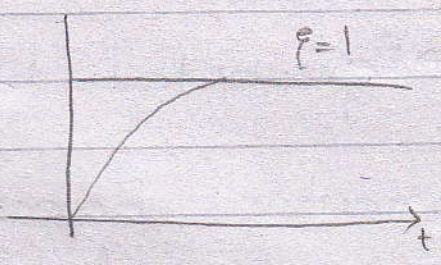
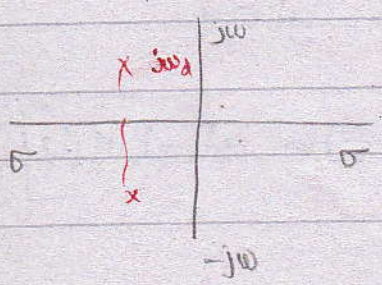
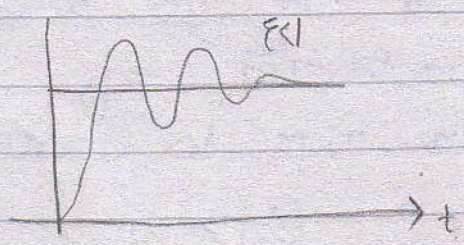
- ↳ The eq obtained by setting the denom of q T/F to zero is called characteristic eq
- ↳ It is imp to analysed the stability of a sys

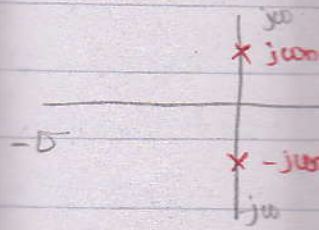
If poles are

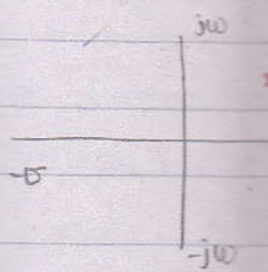
+ve ⟶ Unstable

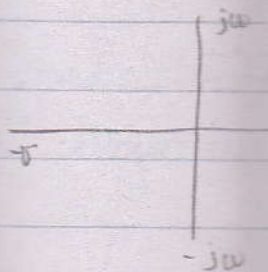
-ve ⟶ stable

⇒ poles represent sys stability whereas zero's gives no information regarding the stability issues it can just shift the response

Location Of Roots	Step Response
	
	
	



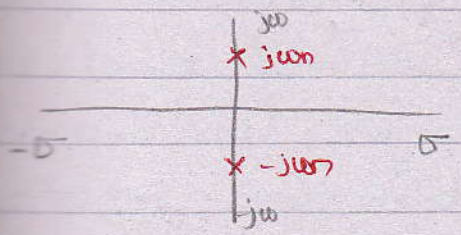


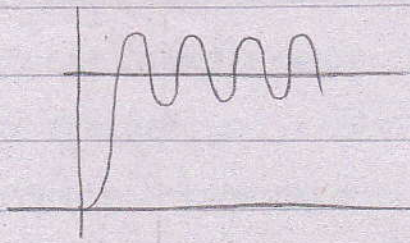


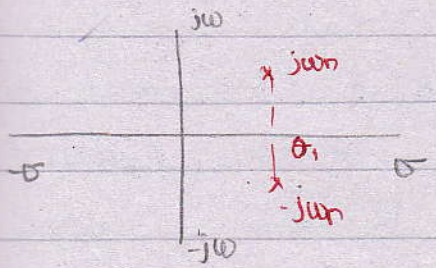
① Real ; -ve

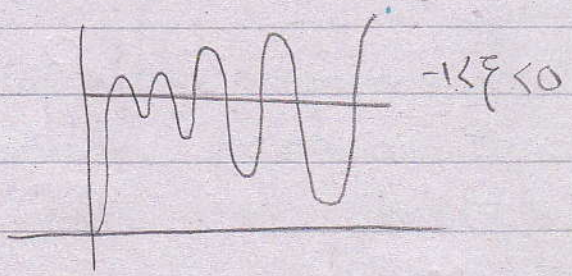
② Real ; -ve

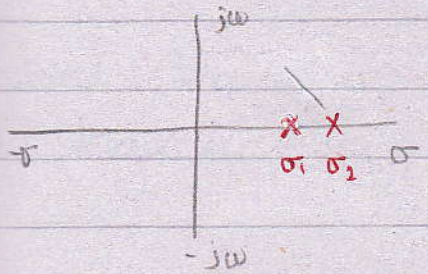
③ Imag

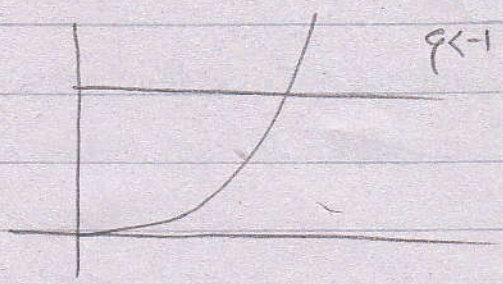








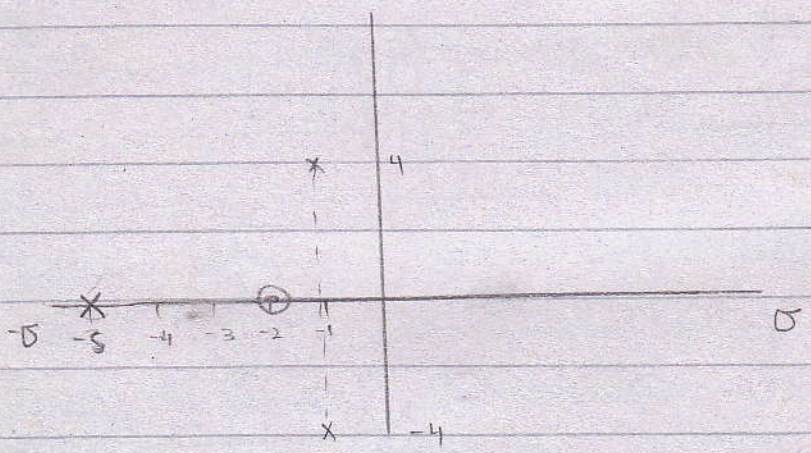




① Real, -ve, distinct

② Real, -ve, equal

③ Imag



MARCH 2014

place transform
tool used for
Differential
classical met
following st

→ The transient
obtained in
classical me
operations &

→ laplace trans
an Algebraic

→ It is then
by simple Al
to obtain t

→ The final
by taking

F(s)

laplace transform is one of the mathematical tool used for the soln of ordinary linear Differential eq. In comparasion with the classical method laplace transform has 2 following superior feature:-

↳ The transient & steady state responses r obtained in one operation. (where as in classical method several math /Arith operations r involved)

↳ laplace transform converts the DE into an Algebric eq in form of S. Where

$$S = \sigma + j\omega$$

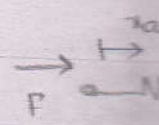
↳ It is then possible to solve the algeb eq by simple Algebric rules. for eg: partial frac to obtain the soln in s domain.

↳ The final soln in time domain is obtained by taking inverse laplace.

$$F(S) = \int_0^\infty f(t)\, e^{-st}\, dt$$

$F(t)$	$F(s)$
$e^{-\sigma t}$	$1/s+\sigma$
$u(t)$	$1/s$
$t\,u(t)$	$1/s^2$
$t^n u(t)$	$n!/s^{n+1}$
$\cos\omega t$	$s/(s^2+\omega^2)$
$\sin\omega t$	$\omega/(s^2+\omega^2)$
$e^{-\sigma t}\sin\omega t$	$\dfrac{\omega}{(s+\sigma)^2+\omega^2}$

* APPLICATION



$\dfrac{x_b(t)}{x_a(t)}$

initial Con

$\Rightarrow \dfrac{X_b(s)}{X_a(s)}$

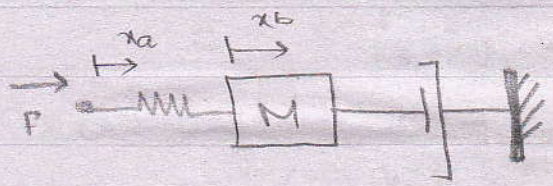
$X_b(s)$

* For particul
and input

Eq ① $\Rightarrow$

Converting in 2

* APPLICATION OF LAPLACE TRANSFORM:



$$\frac{x_b(t)}{x_a(t)} = \frac{k}{MD^2+BD+k}$$

(Initial Conds are zero i.e sys initially @ rest)

$$\Rightarrow \frac{X_b(s)}{X_a(s)} = \frac{k}{Ms^2+Bs+k}$$

$$= \frac{k}{M\left(s^2+\frac{B}{M}s+\frac{k}{M}\right)}$$

$$X_b(s) = \frac{k/M}{s^2+\frac{B}{M}s+\frac{k}{M}} \cdot X_a(s) \qquad (1)$$

* For particular values of parameters (M, B & k) and input $x_a(s)$

Eq (1) $\Rightarrow$ $X_b(s) = \frac{s+2}{s(s+1)(s+3)}$

Converting into Partial frac

$$X_b(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

Now

$$A = \frac{s+2}{s(s+1)(s+3)} \cdot s = \frac{2}{3}$$

$$B = \frac{(s+1)(s+2)}{s(s+1)(s+3)} = \frac{1}{(-1)(2)} = -0.5$$

$$C = \frac{(s+3)\ s+2}{s(s+1)(s+3)} = \frac{-1}{(-3)(-2)} = -\frac{1}{6}$$

Hence

$$X_b(s) = \frac{2}{3s} - \frac{1}{2(s+1)} - \frac{1}{6(s+3)}$$

Taking In laplace

$$\Rightarrow x_b(t) = \left[\frac{2}{3} - \frac{1}{2}e^{-t} - \frac{1}{6}e^{-3t}\right]u(t)$$

S.S Response

TRANSIENT Response.

*mentioning of u(t) with the last 2 terms is optional here since in Question it is not mentioned that we r using step i/p if mention then use u(t) with all terms

Consi

$\Rightarrow$

$X_b(s)$

then

$A =$

$B =$

In po

$A =$

$B =$

(2)

$$X_b(s) = \frac{s+2}{s^2+2s+2}$$

Considering $s^2+2s+2 = 0$

$$s = -1 \pm j$$

$$\Rightarrow X_b(s) = \frac{s+2}{(s+1-j)(s+1+j)}$$

$$X_b(s) = \frac{A}{(s+1-j)} + \frac{B}{(s+1+j)}$$

then

$$A = \left.\frac{s+2}{s+1+j}\right|_{s=-1+j} = \frac{-1+j+2}{-1+j+1+j} = \frac{1+j}{2j} = \frac{-1}{2}j + \frac{1}{2}$$

$$B = \left.\frac{s+2}{s+1-j}\right|_{s=-1-j} = \frac{-1-j+2}{-1-j+1-j} = \frac{1-j}{-2j} = \frac{1}{2}j + \frac{1}{2}$$

In polar form

is

$A = 0.707 \angle 135°$

i/p

$B = 0.707 \angle 45°$

ame

Using Eq (from Chap 2)

$$\Rightarrow x_b(t) = 2A_0 e^{-\sigma t} \sin(\omega_d t + \psi + 90^\circ)$$

$$= 2(0.707) e^{-t} \sin(1t + 45^\circ + 90^\circ)$$
$$= 1.414\, e^{-t} \sin(t + 135^\circ)$$

Where

$\omega_d = 1$ (from poles)

$\psi = 45^\circ$

$\sigma = 1$

$A_0 = 0.707$

4.11 $(D^2+2D+2)(D+4)\, x(t) = (D+2)\, f(t)$

When

a) $f(t) = \delta(t)$

b) $f(t) = 10u(t)$

c) $f(t) = tu(t)$

d) $f(t) = t^2 u(t)$

$$[D^3 + 6D^2 + 10D + 8]\, x(t) = (D+2)\, f(t) \quad \rightarrow (A)$$

THEOREM:

REAL TIME DIFFERENTIATION THEOREM [R.T.D.T]

$$L[Df(t)] = sF(s) - f(0)$$

Now, App

$[s^3 x$

Sys trans

$\frac{x(t)}{f(t)}$

$\frac{X(s)}{F(s)}$

For a

$$L\{D^2f(t)\} = s^2F(s) - s f(0) - Df(0)$$

$$L\{D^n f(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} Df(0) \cdots$$
$$\cdots\cdots s D^{n-2} f(0) - D^{n-1} f(0).$$

Now, Applying RTDT on eq (A)

$$[s^3 X(s) - s^2 X(0) - sDx(0) - D^2 x(0)] +$$
$$6[s^2 x(s) - sx(0) - Dx(0)] + 10[sx(s) - x(0)]$$
$$+ 8X(s) = [sF(s) - F(0)] + 2F(s)$$

Sys transfer funct

$$\frac{x(t)}{f(t)} = \frac{D+2}{(D+4)(D^2+2D+2)}$$

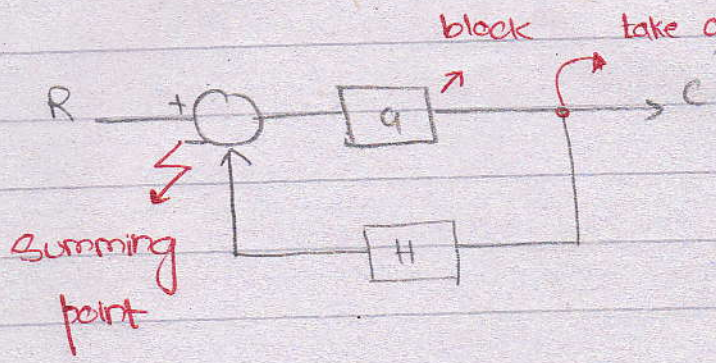
$$\frac{X(s)}{F(s)} = \frac{s+2}{(s+4)(s^2+2s+2)}$$

$$= F(s) \frac{s+2}{(s+4)(s+1+j)(s+1-j)} \rightarrow (B)$$

For a

System Representation

let us consider a closed loop sys

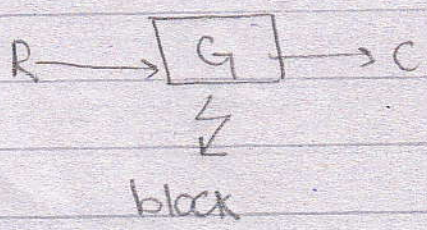


A point from whe copy of the coming signal can be taken out & given another sys.

This form is called as Canonical Form

Where

R = input
C = output
G = Forward transfer func
H = Feedbck transfer "
E = Actuating signal / error signal
B = Feedbck signal
C/R = closed loop t/f or control ratio
E/R = Error ratio
B/R = Feedbck "
GH = open loop t/f



C = GR

From Fig 1.

C =
E
B

For finding

For

+ve f
-ve f

From Fig 1:

$$C = GE \quad \text{—— (1)}$$
$$E = R \mp B \quad \text{—— (2)}$$
$$B = HC \quad \text{—— (3)}$$

For finding C/R:-

$$C = GE$$
$$C = G(R \mp B)$$
$$C = GR \mp GB$$
$$C = GR \mp GHC$$
$$C \pm GHC = GR$$
$$C(1 \pm GH) = GR$$
$$\frac{C}{R} = \frac{G}{1 \pm GH}$$

For

-ve fb, $1 - GH$ (in denominator)

-ve fb, $1 + GH$ (in numerator)

$\frac{E}{R} = ?$

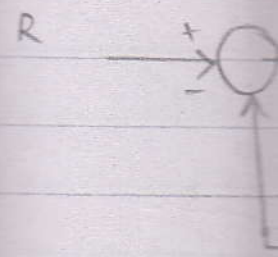
$$E = R \mp B$$
$$= R \mp HC$$
$$= R \mp HGE$$
$$E \pm HGE = R$$
$$E(1 \pm GH) = R$$
$$\frac{E}{R} = \frac{1}{1 \pm GH}$$

$\frac{B}{R} = ?$

$$E = R \mp B$$
$$\frac{C}{G} = R \mp B$$
$$\frac{B/H}{G} = R \mp B$$
$$\frac{B}{GH} = R \mp B$$
$$\frac{B}{GH} \pm B = R$$
$$B\left(\frac{1}{GH} \pm 1\right) = R$$
$$\frac{B}{R} = \frac{GH}{1 \pm GH}$$

STEPS

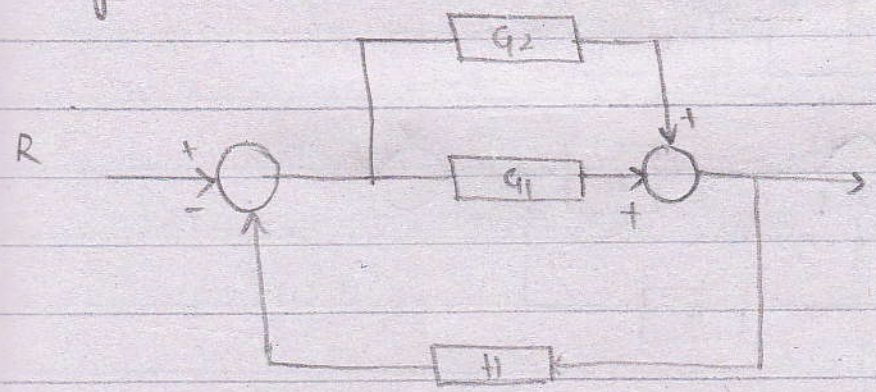
1) Combine
2) Combine
3) Elimina
4) Shift
take
majou
5) Repeat
Form is
6) Repeat
(Used only
sys)



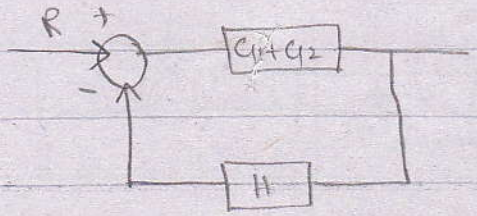
Applying the

Steps For Block Diagram Reduction

1) Combine all cascade blocks (theorem 1)
2) Combine all parallel blocks (" 2)
3) Eliminate " minor feedback loops (" 4)
4) Shift summing point to the left & take off point to the right of the major loop. (theorem 7, 10 and 12)
5) Repeat step 1-4 until Canonical form is achieved for a particular i/p.
6) Repeat step 1-5 for each i/p as req. (Used only when xle inputs are thr in a sys)



Applying theorem 2:

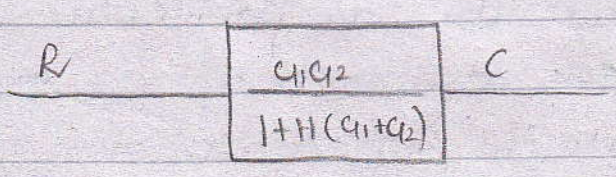


By seeing arrows direction we decide wch theorem should be applied.

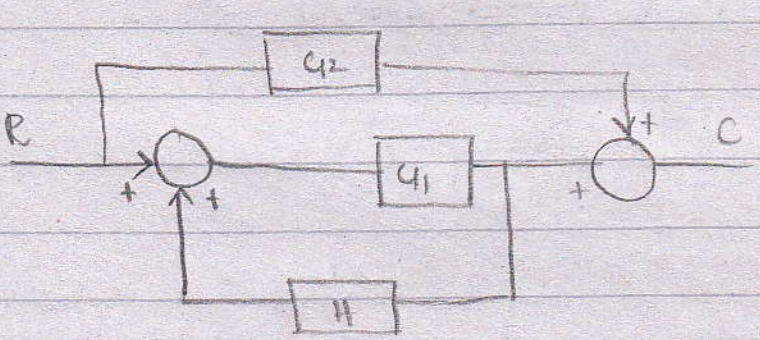
i/p to i/p arrows → Theorem 2

o/p to i/p " → " 4

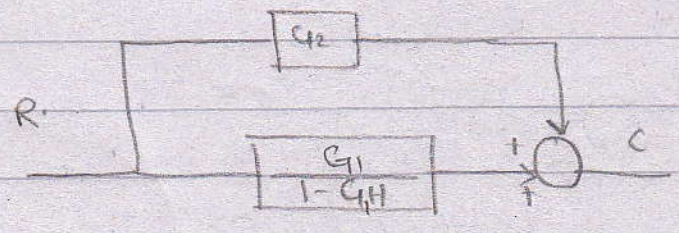
*APPLYING THEOREM 4:



(2)

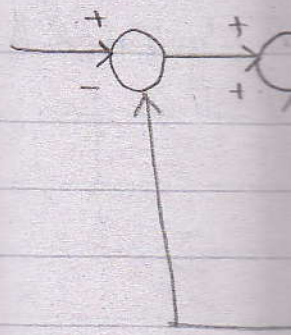


APPLYING THEOREM 4

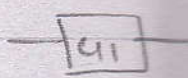


R

(3)

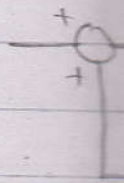


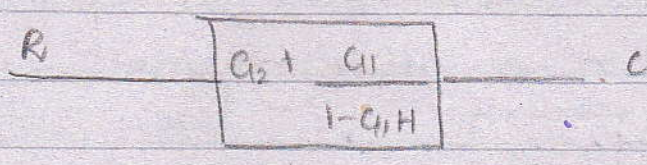
Applying



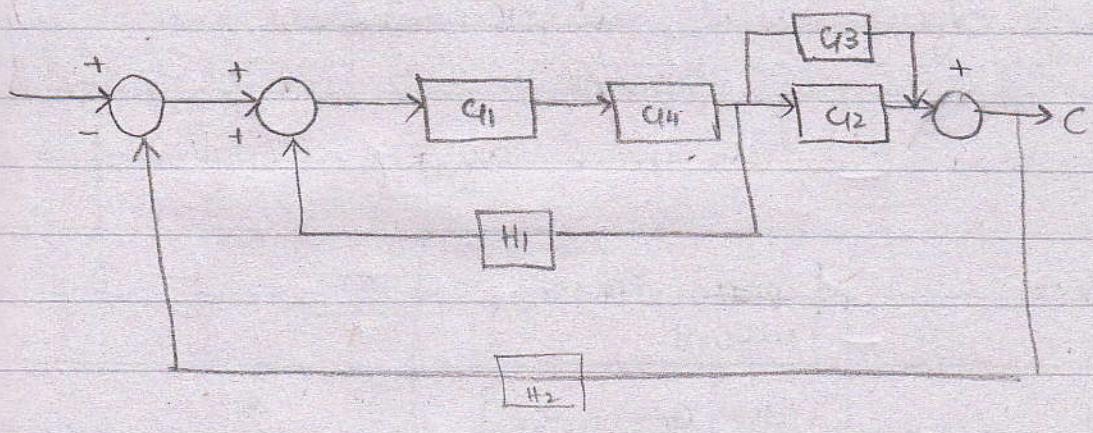
Applyin

Apply



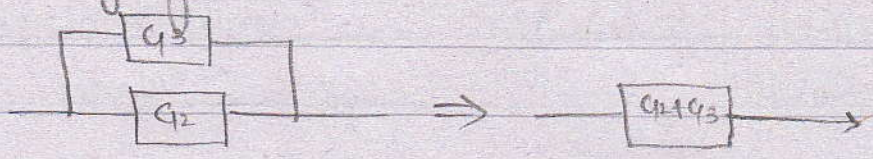


(3)

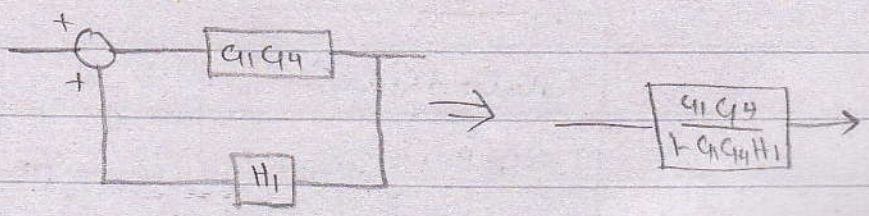


Applying theorem 1

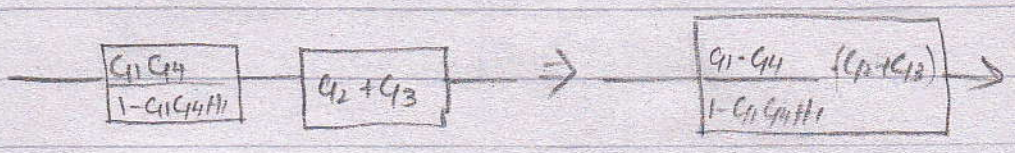
Applying theorem 2



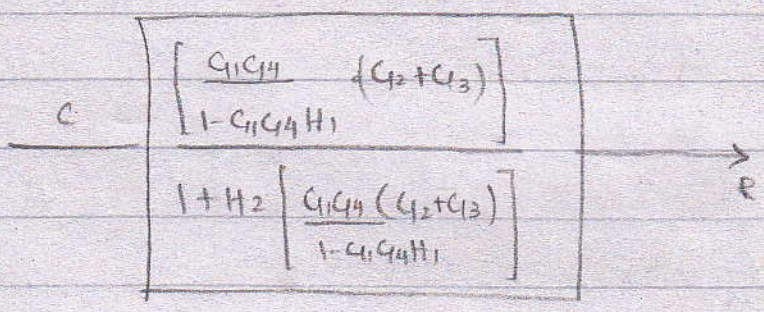
Applying theorem 4



Applying theorem 1:



Applying theorem 4



Solving this for C

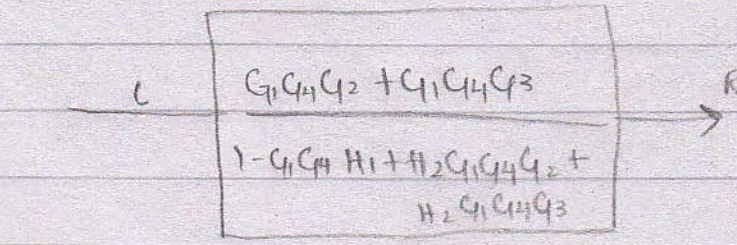
Denominator

$$\frac{G_1G_4}{1-G_1G_4H_1}\cdot(G_2+G_3)$$

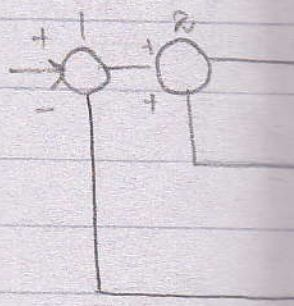
Numerator

$$\frac{1-G_1G_4H_1+H_2G_1G_4G_2+H_2G_1G_4G_3}{1-G_1G_4G_1}$$

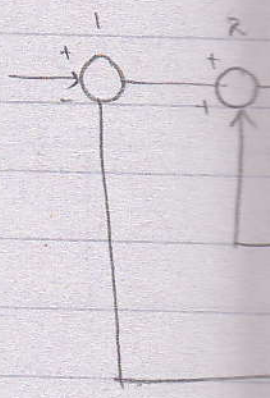
$$\frac{G_1G_4G_2+G_1G_4G_3}{1-G_1G_4H_1}$$



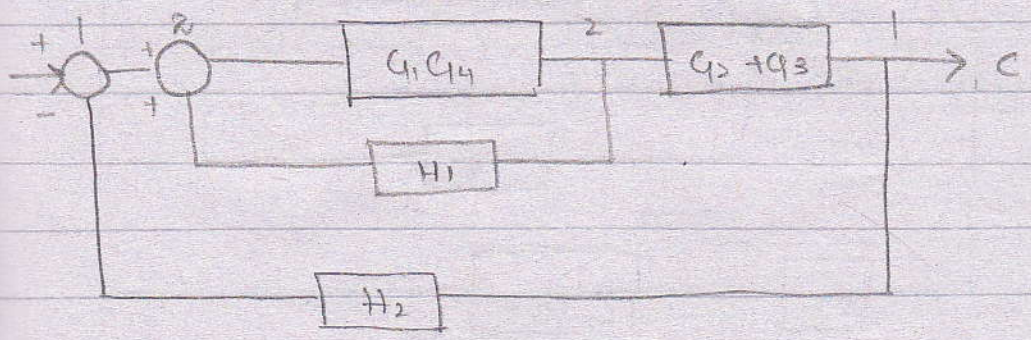
Simplify th
by isol



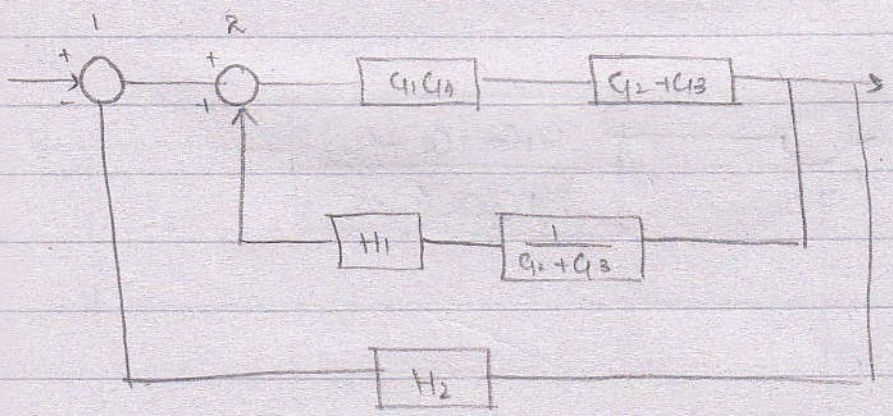
Applying



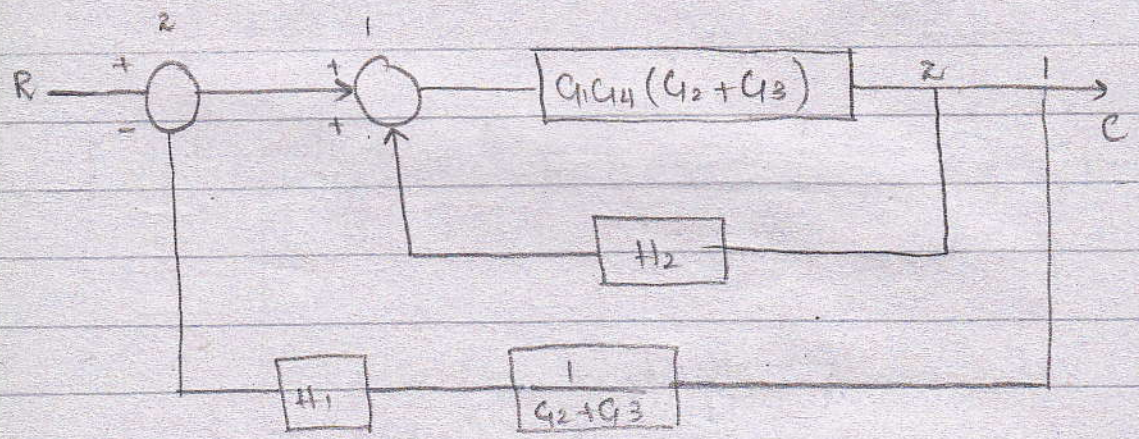
Simplify the block diagram given in q.3 by isolating H_1 as a feedback loop.



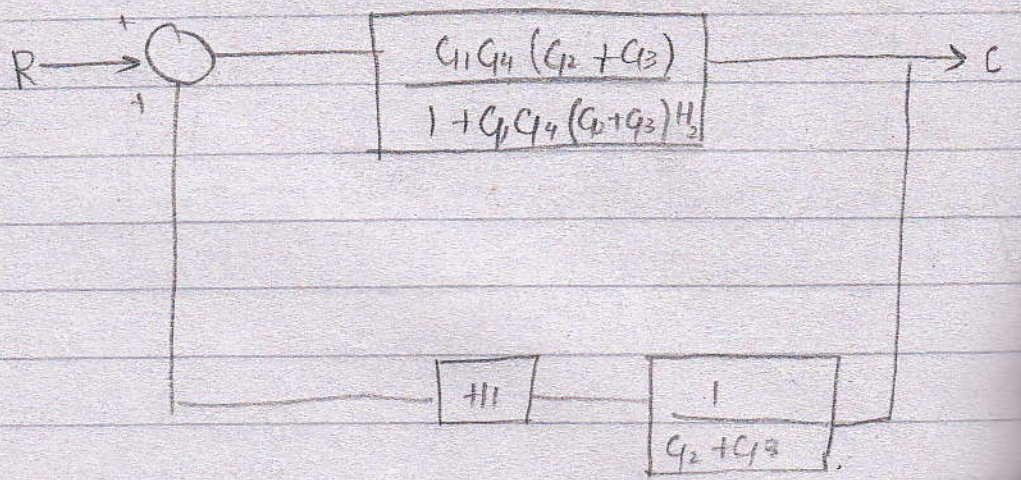
Applying theorem 10



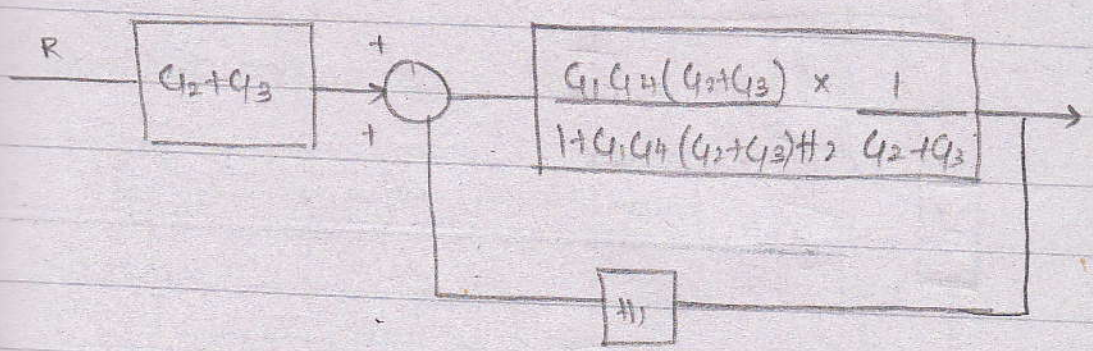
Applying theorem 6

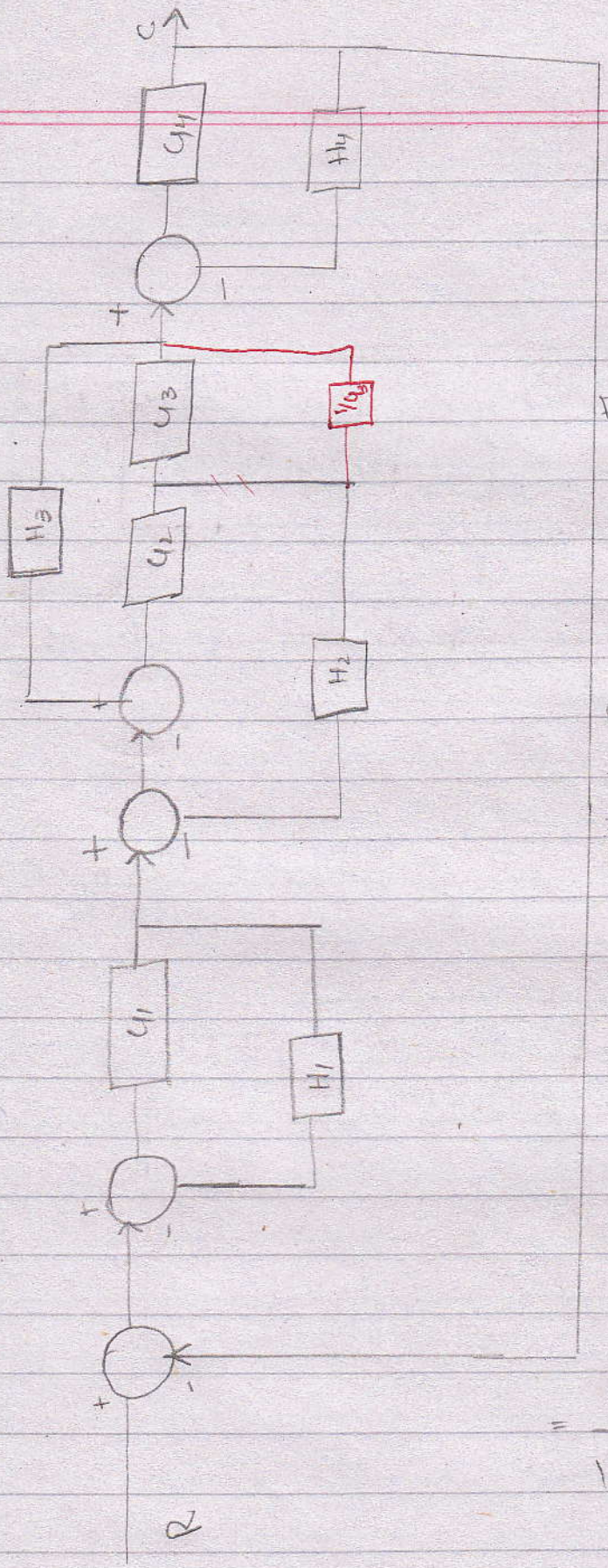


Applying theorem 4.



Applying theorem 5:





$* (G_2 * G_3) \| (H_2 * \frac{1}{G_3})$

Using theorem 6.

From theorem 4

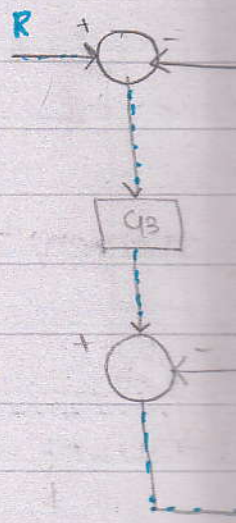
$$\frac{G_2 G_3}{1 + G_2 G_3 \frac{H_2}{G_3}}$$

$$= \frac{G_2 G_3}{1 + G_2 H_2}$$

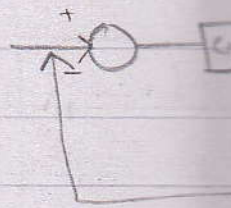
from theorem 4

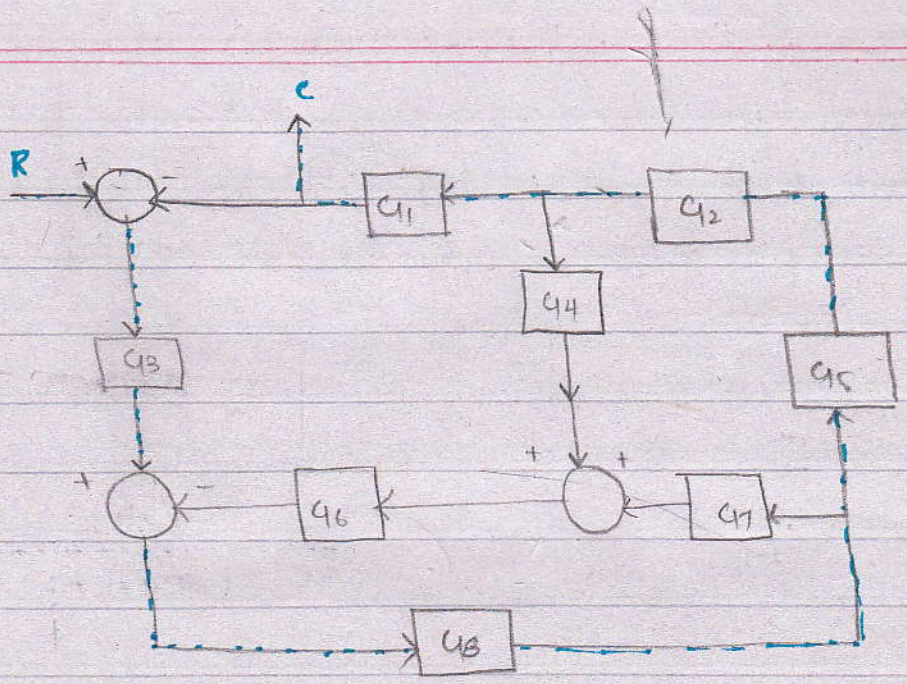
$$\frac{G_2 G_3 / (1 + G_2 H_2 H_3)}{1 + \frac{G_2 G_3}{1 + G_2 H_2} \cdot H_3}$$

$$= \frac{G_1 G_3 / (1 + G_2 G_3 H_3)}{1 + \frac{G_2 G_3}{1 + G_3 G_3 H_3} \cdot \frac{H_2}{G_3}}$$

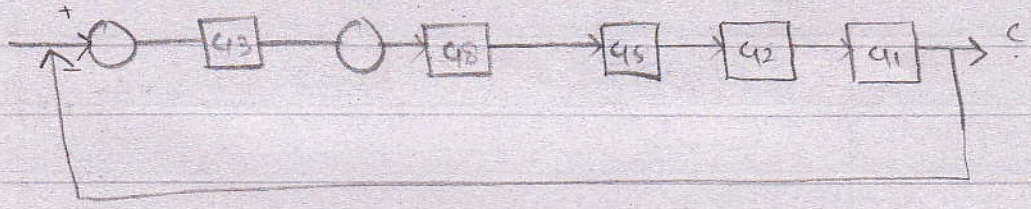


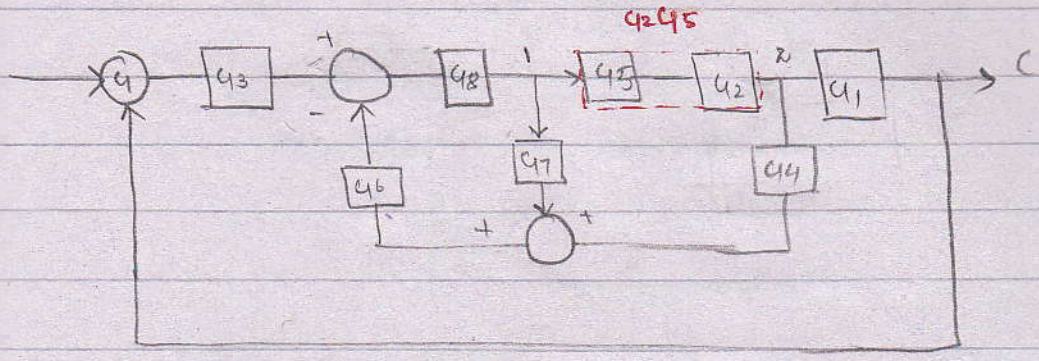
* R to





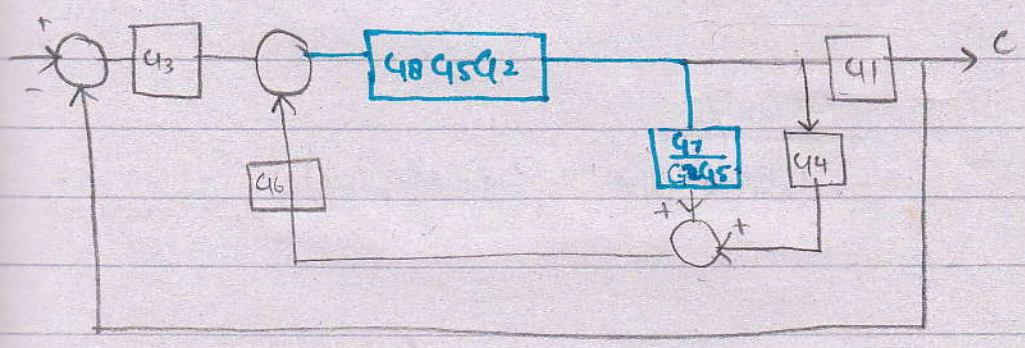
* R to C





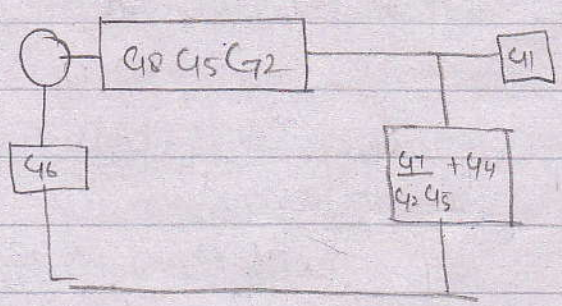
* Theorem 1

* Theorem 10

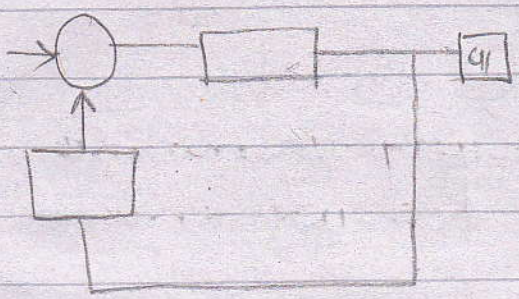


Theorem 2 :

$$\frac{G_7}{G_2G_5} + G_4$$



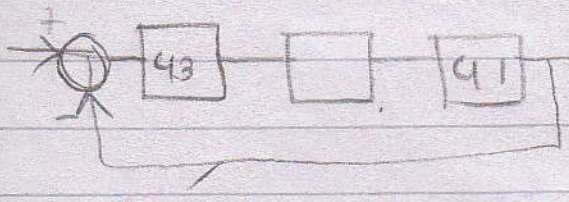
$$\Rightarrow \left(\frac{G_7}{G_2G_5} + G_4\right) G_6 = \frac{G_6G_7}{G_2G_5} + G_4G_6$$



$$\frac{G_8 G_2 G_5}{1+(G_8 G_2 G_5)\left[\frac{G_6 G_7}{G_2 G_5}+G_4 G_5\right]}$$

$$= \frac{G_8 G_2 G_5}{1+G_8 G_2 G_5\left(\frac{G_6 G_7 + G_2 G_4 G_5 G_6}{G_2 G_5}\right)}$$

$$= \frac{G_8 G_2 G_5}{1+G_8(G_6 G_7 + G_2 G_4 G_5 G_6)}$$



$$= \frac{G_1 G_2 G_3 G_5 G_8}{1+G_8(G_7 G_6 + G_2 G_4 G_5 G_6)}$$

$$= \frac{G_1 G_2 G_3 G_5 G_8 \,/\, 1+G_8(G_7 G_6 + G_2 G_4 G_5 G_6)}{1+\left[\frac{1+G_1 G_2 G_3 G_5 G_8}{1+G_8(G_7 G_6 + G_2 G_4 G_5 G_6)}\right]}$$

$$\frac{C}{R} = \frac{G_1G_2G_3G_5G_0}{1+G_6G_7G_8+G_2G_4G_5G_6G_0 + G_1G_2G_3G_5G_0}$$

for numerator
Consider the forward FB

for denom
add all backward FB.

Signal Flow Graph (SFGs)

It is the advanced version of blk diag technique by SJ Mason. This technique is used to represent a sys graphically with the help of algebric eq's. wch a written ~~by~~ in the form of cause & their effect.

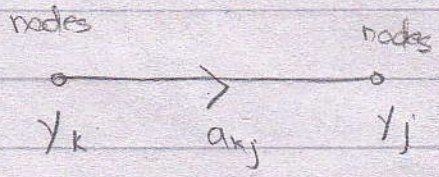
$$y_j = \sum_{k=1}^{N} a_{kj}\, y_k$$

$$j(\text{effect}) = \sum [\text{gain from } k \to j] * [k^{th} \text{ cause}]$$

$$\text{Output} = \sum [\text{gain}][\text{input}]$$

REPR:

$$y_j = a_{kj}\, y_k$$



The de
systems
diff eq
laplace

After the
find the

ELEMENT

*NODES:
nodes r
variables

The num
line se
Each b
branch
represen

Branch
Amplifier

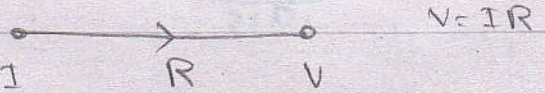
The technique is applicable for linear systems only. The given integro-diff eq of the sys is converted into laplace transform eq (Algebric eq)

After that SFGs can be used to find the sys transfer func.

ELEMENT OF SFG's:

*NODES:

nodes r used to represent sys variables.



The num of nodes r connected by a line segment called branches.
Each branch is associated with a branch gain & arrow. The arrow represents the flow of info

Branch is also known as Unilateral Amplifier.

PROBLEM:

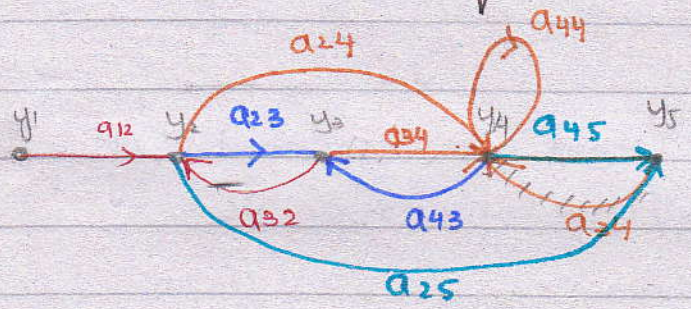
$$y_2 = a_{12} y_1 + a_{32} y_3 \quad *$$

$$y_3 = a_{23} y_2 + a_{43} y_4 \quad *$$

$$y_4 = a_{24} y_2 + a_{34} y_3 + a_{44} y_4 \quad *$$

$$y_5 = a_{25} y_2 + a_{45} y_4 \quad *$$

Draw SFG from above eqs.



A node with all outgoing branches is called i/p for source node

eg

y_1 is the i/p load.

A " " " incoming "

" " o/p or sink node.

eg

* PATH:

A path is
branches

Forward p
i/p load
such a
more than

* I/p Node
* o/p nod

* " "

* " "

* " "

*PATH:

A path is a continous succession of branches moving in the same direction.

Forward path is the path that starts from i/p load and ends at o/p load in such a way that no load is traversed more than once.

* I/P Node = 1
* o/p node is y_2

$y_1 - y_2$

* " " " y_3

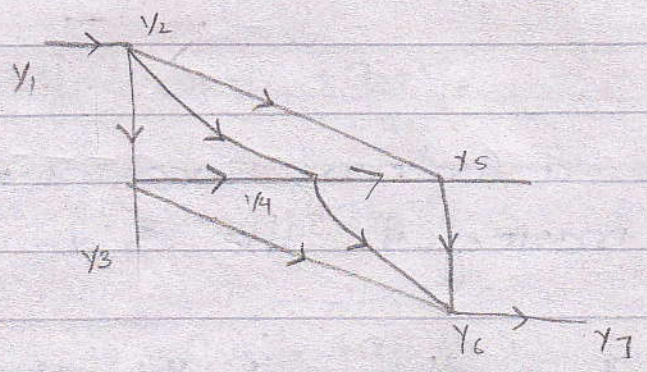
$y_1 - y_2 - y_3$
$y_1 - y_2 - y_4 - y_3$

* " " " y_4

$y_1 - y_2 - y_3 - y_4$
$y_1 - y_2 - y_4$

* " " " y_5

$y_1 - y_2 - y_3 - y_4 - y_5$
$y_1 - y_2 - y_5$
$y_1 - y_2 - y_3 - y_5$



No of paths formed = 6

i) $y_1 - y_2 - y_3 - y_6 - y_7$

ii) $y_1 - y_2 - y_4 - y_6 - y_7$

iii) $y_1 - y_2 - y_5 - y_6 - y_7$

iv) $y_1 - y_2 - y_4 - y_6 - y_7$

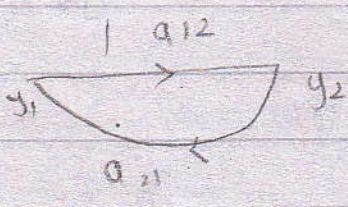
v) $y_1 - y_2 - y_4 - y_5 - y_6 - y_7$

vi) $y_1 - y_2 - y_3 - y_4 - y_6 - y_7$

Loop

A loop is a path that starts & ends at same node & along with no node is repeated more than once.

* $y_1 - y_2 - y_1$



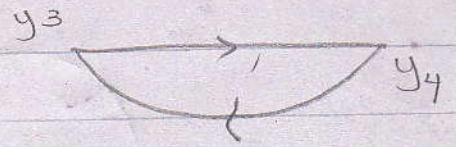
* $y_3 - y_4$

* $y_4 - y_4$

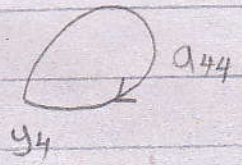
* $y_2 - y_4$

PROBLEM:-

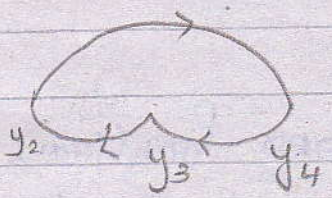
* $y_3 - y_4 - y_3$



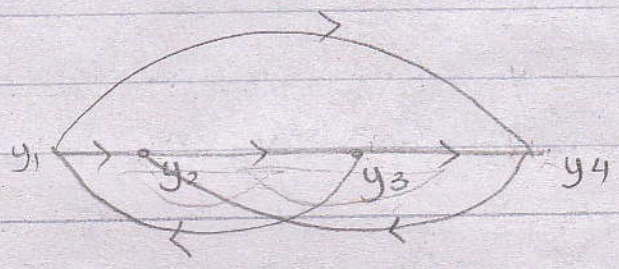
* $y_4 - y_4$



* $y_2 - y_4 - y_3 - y_2$



PROBLEM:-

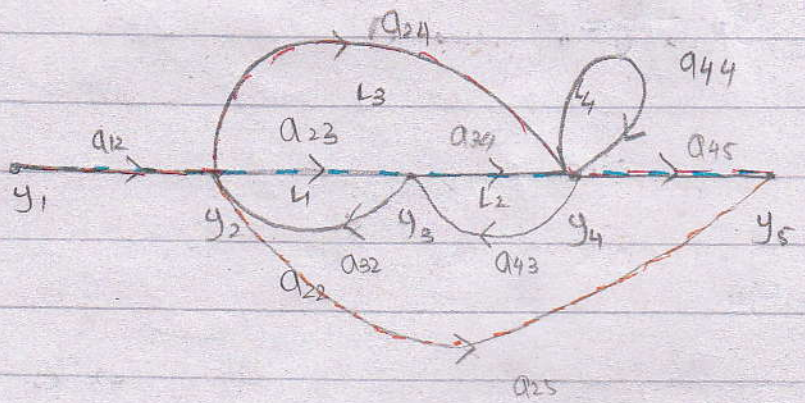


$y_1 - y_R - y_3 - y_1$

$y_2 - y_3 - 44 - y_2$

$y_1 - y_4 - y_2 - y_3 - y_1$

28th March 2014



Forward Path Gain:

A product of branch gain of forward path is called forward path gain. Represented by M.

$$M_1 = a_{12}\ a_{23}\ a_{34}\ a_{45}$$

$$M_2 = a_{12}\ a_{25}$$

$$M_3 = a_{12}\ a_{24}\ a_{45}$$

Loop Gain:-

The product of branch gain of a loop is called loop gain. represented by L or P

$$L_1 = a_{23}\ a_{32}$$

$$L_2 = a_{34}\ a_{43}$$

Where

$L_3 = a_{24}\ a_{43}\ a_{32}$

$L_4 = a_{44}$

*

MASON GAIN FORMULA → To find $\frac{C}{R}$

$$G = \frac{\%\ out}{\%\ in} = \sum_{k=1}^{N} \frac{M_k \Delta_k}{\Delta}$$

Where

Y_1 Y_{in} = input node variable

Y_5 Y_{out} = output " "

G = gain b/w Y_{in} & Y_{out}

4 N = total no of forward paths b/w Y_{in} and Y_{out}.

3 M_k = gain of kth forward path b/w Y_{in} and Y_{out}

$\Delta = 1 -$ (Sum of the gain of All indiv loops)

+ (sum of the products of all possible comb of 2 non-touching loop)

− (sum of the prod of all " " " 3 " ")

Δ_k = the Δ for the part of SFG ie no touching with k^{th} forward path.

Now

Considering previous problem

$$q = \frac{Y_5}{Y_1} = \frac{(M_1\Delta_1 + M_2\Delta_2 + M_3\Delta_3)}{\Delta} \rightarrow (A)$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_4) - 0$$
$$= 1 - L_1 - L_2 - L_3 - L_4 + L_1L_4$$

Eg

$\Delta_1 = 1$ *

$\Delta_2 = 1 - (L_2 + L_4)$ * *While considering path M_2 L_2 & L_4 are Non-touching. $\therefore$ L_2 & L_4 are touching so if non-touching then *

$\Delta_3 = 1$ *

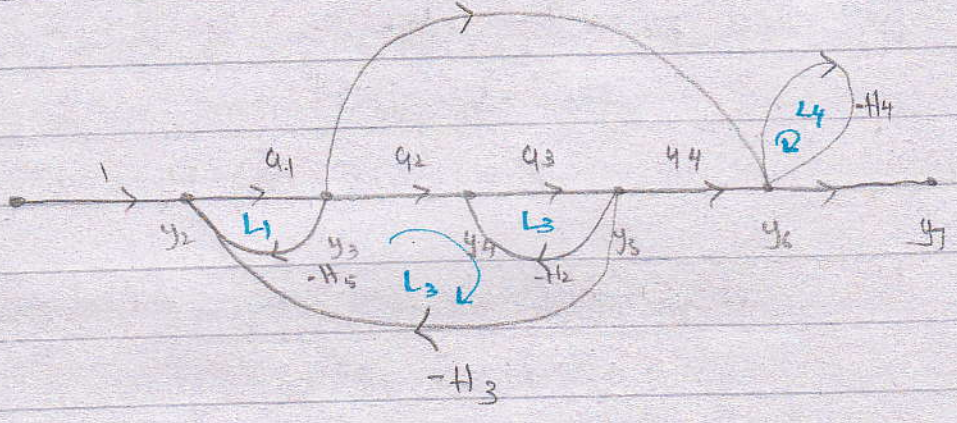
Now

$$\text{A} \Rightarrow \quad q = \frac{a_{12}a_{23}a_{34}a_{45} + a_{12}a_{25}[1-L_2-L_4] + a_{12}a_{24}a_{45}}{1-(L_1+L_2+L_3+L_4)+L_4L_4}$$

$$= \frac{a_{12}[a_{23}a_{34}a_{45} + a_{25}(1-L_2-L_4) + a_{24}a_{45}]}{1-(L_1+L_2+L_3+L_4)+L_4L_2}$$

$$= \frac{a_{12}[a_{23}a_{34}a_{45} + a_{25}(1-a_{34}a_{43}-a_{44}) + a_{24}a_{45}]}{1-(a_{23}a_{32}+a_{34}a_{43}+a_{24}a_{43}a_{32}+a_{44})+a_{23}a_{32}a_{34}a_{43}}$$

And.



* There are 2 forward paths.

$$C_1 = \frac{Y_7}{Y_1} = \frac{M_1\Delta_1 + M_2\Delta_2}{\Delta} \rightarrow ①$$

then,

$$M_1 = G_1 G_2 G_3 G_4$$

$$M_2 = G_1 G_5$$

and loop gains are

$$L_1 = -G_1H_1 \qquad L_3 = -G_1G_2G_3H_3$$

$$L_2 = -G_3H_6 \qquad L_4 = -H_4$$

$$\Delta = 1-(L_1+L_2+L_3+L_4)+(L_1L_2+L_1L_4+L_2L_4+L_3L_4)-L_1L_2L_4$$

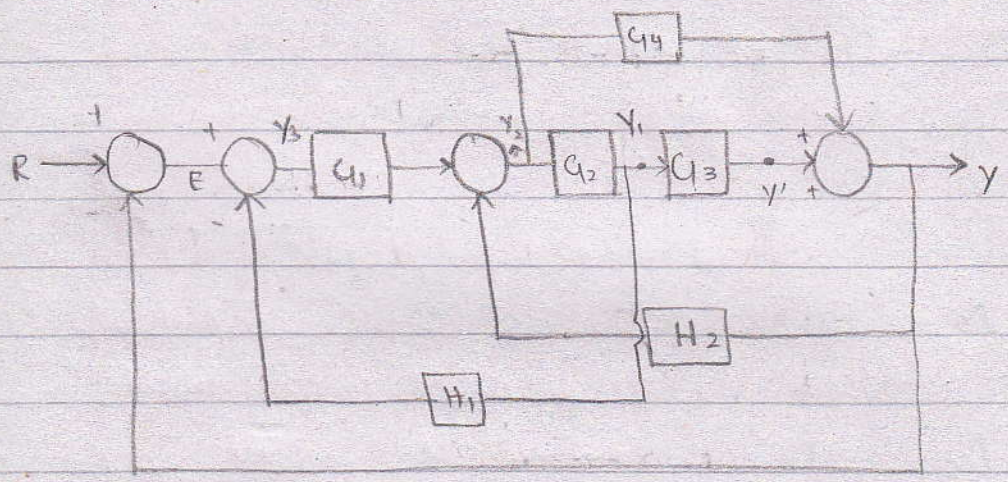
and

$$\Delta = 1-0+0 = 1$$

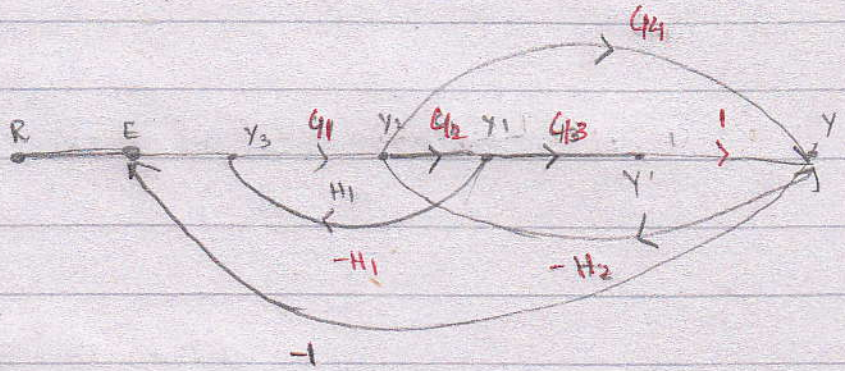
$$\Delta_2 = 1-L_2$$

$$① \Rightarrow C_4 = \frac{C_1C_2C_3C_4 + C_1C_5(1+C_3H_6)}{\Delta}$$

$$= \frac{C_1C_2C_3C_4 + C_1C_5 + C_1C_5C_3H_6}{\Delta}$$



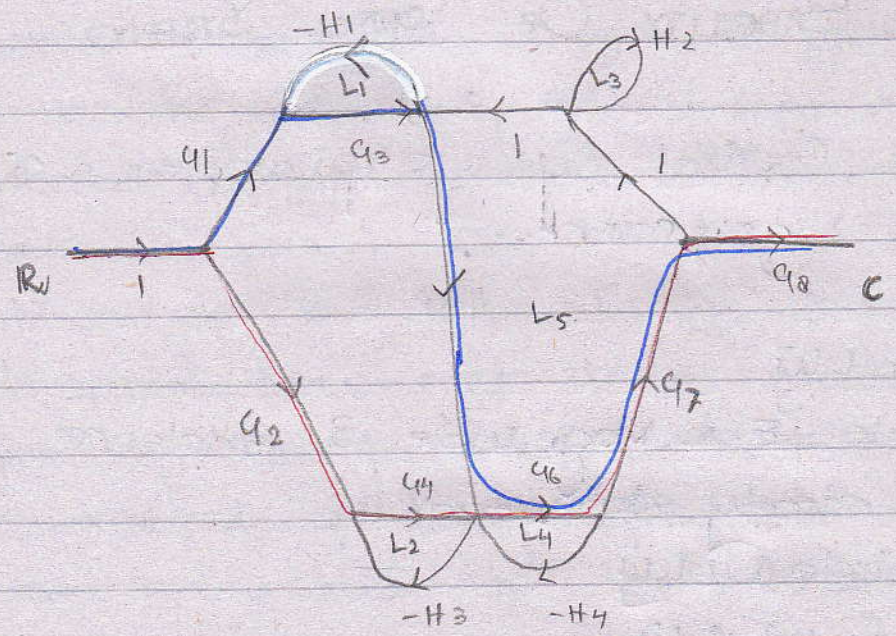
SFG



$M_1 = G_1$

$M_2 = G_2$

$\Delta = 1 -$



* Forward path 1.

* " " 2

$M_1 = G_1 G_3 G_5 G_6 G_7 G_8$

$M_2 = G_2 G_4 G_6 G_7 G_8$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_1L_2 + L_1L_3 + L_1L_4 + L_2L_3 + L_3L_4)$

$\Delta_1 = 1$

$\Delta_2 = 1 - (L_1 + L_3)$

31st march

Stability Of Control System

for a Control sys, 5 parameters r of great significance.

1) Stability
2) ~~State Error~~ Magnitude & existence of steady state error
3) Controlability
4) Observability
5) parameter sensitivity

For the analyses & designed purpose stability can be classified in2:-

→ absolute stability
→ Relative "

Absolute Stability:

It refers 2 the Conditions that the sys is stable or unstable. The ans is simply yes or no

Relative S

It is the
stabilit
sys is
It is o
to find
for eg
Overshoo
Used in
response

The stab
from the
eg. let
characte

$b_n s^n + b_n$

1: All Coeff
positive

R: No term
characteristic

Relative Stability

It is the majow of the degree of stability wch means once the sys is found 2 be stable it is of the interest of designer to find how mch it is stable for eg ; the parameters such as overshoot & damping ratio & used in relation to the transient response indicate the relative stability

Necessary Conditions for Stability.

The stability of the sys can be obt from the coeff of its characteristic eq. let us consider the sys with characteristic eq

$$b_n s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0 = 0 \qquad b_0 > 0$$

1: All coeff of characteristics eq shd be positive

2: No term of 's' shd be missing in the characteristic eq.

Conditions ① and ② are necessary for a sys to be stable if sum of coeff is zero or neg, then

it can be concluded that the sys is not stable

If all coeff are +ve & no 's' term is missing then there's a probability that the sys may be stable wch will be examined by sufficient cond of stability (By Routh Hurwitz Criteria

a) $s^5 + 4s^4 + s^2 + s + 2 = 0$

b) $5s^4 - 2s^3 + 3s^2 + 2s + 16 = 0$

c) $12s^3 - s + 10 = 0$

All are Unstable

ROUTH

It is
Sys
finding
erstic

The
wch
LTI sys
Criteria

i) Whether
eq lie
ii) The
the in
right
also

Routh Hurwitz Criteria:

It is the method to determine sys stability of LTI sys. without finding the roots of the charact-eristic eq.

The Criteria is the algebraic method wch uses characteristic eq of LTI sys with const coeff. The Criteria tests

i) Whether any roots of the characteristic eq lie in the right half of s-plane
ii) The no of roots that lie on the imaginary axis & in the right half of the s plane is also indicated.

Routh Array:

Consider a characteristic eq

$$b_6 S^6 + b_5 S^5 + b_4 S^4 + b_3 S^3 + b_2 S^2 + b_1 S + b_0 = 0$$

S^6	b_6	b_4	b_2	b_0
S^5	b_5	b_3	b_1	—
S^4	C_1: $\frac{b_5 b_4 - b_3 b_6}{b_5}$	C_2: $\frac{b_5 b_2 - b_1 b_6}{b_5}$	C_3: $\frac{b_5 b_0}{b_5}$	
S^3	D_1: $\frac{C_1 b_3 - C_2 b_5}{C_1}$	D_2: $\frac{C_1 b_1 - C_3 b_5}{C_1}$		
S^2	E_1: $\frac{D_1 C_2 - D_2 C_1}{D_1}$	E_2: $\frac{\cancel{D_1} C_3}{\cancel{D_1}}$		
S^1	F_1: $\frac{E_1 D_2 - D_1 E_2}{E_1}$	—		
S^0	$\frac{\cancel{F_1} E_2}{\cancel{F_1}}$			

Once the routh Array is Completed, then the next step is to investigate the no of signs changes in the first column of tabulation.

* The roots of the characteristic eq are all in the left half of s-plane. if all the elements of the first column have the same sign.

* If there are changes of sign in the elements of first column the no of sign changes indicates the no of roots with the real part.

eg

$(s-2)(s+1)(s-3) = 0$ → Unstable

$(s^2+s-2s-2)(s-3) = 0$

$(s^2-s-2)(s-3) = 0$

$(s^3-3s^2-s^2+3s-2s+6) = 0$

$s^3 - 4s^2 + s + 6 = 0$

s^3 1 1

s^2 -4 6

s^1 $\frac{-4-6}{-4} = \frac{5}{2} = 2.5$

s^0 $\frac{30/2}{5/2} = 6$

3RD APRIL 2014

② $2s^4 + s^3 + 3s^2 + 5s + 10 = 0$

→ system is Unstable
→ stability of the sys is determined by routh Array

s_4	2	3	10
s_3	1	5	
s^2	$\frac{3-10}{1} = -7$	$\frac{10-0}{1} = 10$	
s^1	$\frac{-35-10}{-7} = \frac{+45}{7}$	0	0
s^0	$\frac{450/7 - 0}{45/7} = 10$		

↳ 1st sign change

2nd sign chang

There r 2 sign changes, so 2 roots lie on the RHS of the plane.

The sys is Unstable

③ $s^6 + 3s^5 +$

PROPERTY

Without
column
÷ded
to red
for the

s^6	1
s^5	~~3~~ 1
s^4	$\frac{2-3}{1} =$
s^3	$\frac{-3-1}{-1} =$ ~~4~~ 1
s^2	$\frac{1+5}{1} = 5$
s^1	$\frac{25-20}{5} =$
s^0	20

③ $S^6 + 3S^5 + 2S^4 + 9S^3 + 15S^2 + 12S + 20 = 0$

PROPERTY

Without changing the sign of first column, any row can be xued or ÷ded by a constant no in order to reduce the size of a row for the convinience of row calc.

S^6	1	2	5	10
S^5	~~3~~ 1	~~9~~ 3	~~12~~ 4	~~0~~ (÷de by 3)
S^4	$\frac{2-3}{1} = -1$	$\frac{5-4}{1} = +1$	$\frac{20}{1} = 20$	0
S^3	$\frac{-3-1}{-1} = \frac{-4}{-1}$ ~~1~~	$\frac{-4-20}{-1} = \frac{-24}{-1}$ ~~5~~	0	0 (÷de by 4)
S^2	$\frac{1+5}{1} = 5$	$\frac{20}{1} = 20$	0	
S^1	$\frac{25-20}{5} = 1$	0		
S^0	20			

Special Cases

Following difficulties might be faced during occasionaly;

- The first element in any row of Routh tabulation is 0, but others are not
- All the elements in one row of routh tabulation are zero.

CASE I:

$$s^4 + s^3 + 2s^2 + 2s + 5 = 0$$

s^4 : 1 2 5

s^3 : 1 2

s^2 : $\frac{2-2}{1} = 0$ $\frac{5}{1} = 5$

s^1 : $\frac{0-5}{0} = \infty$ → test break

s^0 :

Remedy #

Replace

$\left(\frac{1}{x}\right)^4 +$

$5x^4$

x^4 5

x^3 2

x^2 $\frac{4-5}{2}$

x^1 $\frac{-1-4}{-1}$

x^0 $\frac{5/2 x^2}{5/2}$

REMEDY

Multiply

where n

Remedy #1:

Replace s by $1/x$

$$\left(\frac{1}{x}\right)^4 + \left(\frac{1}{x}\right)^3 + 2\left(\frac{1}{x}\right)^2 + 2\left(\frac{1}{x}\right) + 5 = 0$$

$$5x^4 + 2x^3 + 2x^2 + x + 1 = 0$$

x^4	5	2	1
x^3	2	1	0
x^2	$\frac{4-5}{2} = \frac{-1}{2} \times 2 = -1$	$\frac{2}{2} = 1 \times 2 = 2$	xly by 2
x^1	$\frac{-1-2}{-1} = \frac{+5}{2}$	0	
x^0	$\frac{5/2 \times 2}{5/2} = 2$		

Remedy #2:

Multiply characteristic eq by $(s+n)$ where n is a natural no ie

$$n = 1, 2, 3, \ldots$$

- Take the lowest value of n.

For $s^4+s^3+2s^2+2s+5=0$

$(s+1)(s^4+s^3+2s^2+2s+5)=0$

$s^5+s^4+2s^3+2s^2+5s+s^4+s^3+2s^2+2s+5=0$

$s^5+2s^4+3s^3+4s^2+7s+5=0$

s^6				
s^5	1	3	7	
s^4	2	4	5	
s^3	$2/2=1 \times 2=2$	$9/2^{x2}=9$	0	xy by 2
s^2	$-5/5=-1$	$5/5=1$		÷de by 5
s^1	$\frac{-11}{-1}=11$	0		
s^0				

- Remedy # 3

Replace 0 by a very small ε.

s^4
s^3
s^2
s^1
s^0

* Case II

When all
Row of
It include
following

1-
2-

Eg: s^5+

S^4	5	2	1
S^3	2	1	0
S^2	ε	5	0
S^1	$\frac{2\varepsilon - 5}{\varepsilon} = \frac{-5}{\varepsilon}$	0	0
S^0	$\frac{-25/\varepsilon - 0}{-5/\varepsilon} = 5$		

* CASE II :

When all the elements in one row of row of routh tabulation are 0, it includes one or more pair of the following conditions :-

1- pair of real roots with two sign
2- " " imaginary roots

Eg: $S^5 + 6S^4 + 8S^3 + 8S^2 + 7S + 4 = 0$

S^5	1	8	7
S^4	4/1	8/2	4/1
S^3	6/1	6/1	
S^2	1	1	0 ⇒ A(s)
S^1	0	0	0 ⇒ test break
S^0			

(Divide by 4)

(Divide by 6)

Remedy:

When a row of 0 occurs, we can perform the following remedy,

1- Take 2/ds of auxillary eq [A(s)]

2- Replace row of 0 with the coeff of the resultant equation

3- Resume the routh test agn.

Auxillary

The eq
the coef
the row

A(s) =

d [A(s
ds

Now

S^2

S^1

S^0

system
change
sys's
sinusoid
with

Auxillary Equation:

The eqn that is formed by using the coeff of the row just above the row of zero.

$$H(s) = s^2 + 1$$

$$\frac{d[A(s)]}{ds} = 2s$$

row of zero indicates that some roots are imaginary

↓

marginally stable

$s^2 + 1 = 0$

$s = \pm j\omega n$

Now

s^2	1	1	0
s^1	2	0	0
s^0	$\frac{2}{2} = 1$		

system is stable bec there is no change in sign (marginally stable).
sys's response is underdamped, sinusoid & sys an oscillator with natural freq 1 rad/sec

Conditional Stability.

$S^3 + 3408.3S^2 + 1204000S + 1.5 \times 10^7 K = 0$

Where

k = gain

Routh stability criteria is best suited to determine the values of k for the sys stability

S^3	1	1204000	
S^2	3408.3	$1.5 \times 10^7 K$	
S^1	$\frac{410.36 \times 10^7 - 1.5 \times 10^7 K}{3408.3}$	0	→ i
S^0	$1.5 \times 10^7 K$		→ ii

For the sys to be stable all roots to be in the left half of the plane, all coeff of first column shd be greater than 0

then

(i) $\Rightarrow \frac{410.36 \times 10^7 - 1.5 \times 10^7 K}{3408.3} > 0$

$K < 273.57$

(iii) $\Rightarrow$ $1.5 \times 10^7 K > 0$

$K > 0$

then

$0 < K < 273.57$

CONCLUSION:

The sys will be stable from $K = 0 - 273.57$

The sys can behave with sustained oscillation if control parameter is designed exactly at $K = 273.57$ wch will give the system's natural freq 1097.27 rad/sec.

$3400.3s^2 + 1.5 \times 10^7 (273.57) = 0$

$s = \pm j\, 1097.27$

When

$\omega_n = 1097.27$ rad/sec.

Steady State Error & Error Efficiency

Bode form of t/f can be written as

$$G(s) = \frac{K_B\, G'(s)}{s^n}$$

Where
the func $G'(s)$ has a steady state gain of unity.

ie $G'(0) = 1$

The term 's^n' in the denominator corresponds to the num of free integrators in the sys. The num 'n' is called the sys type number.

* Types Of Sys:

$$G(s) = \frac{k(1 + b_1 s + b_2 s^2 + \cdots b_w s^w)}{s^n(1 + a_1 s + a_2 s^2 + \cdots a_4 s^4)} \quad \text{(1)}$$

$s^n \longrightarrow$ determines Error

Type Zero
(no p

eg:

Type #1

eg:

Type #2

eg:

Any sys following

R

Type Zero System:

(no poles at Origin)

eg:

$$G(s) = \frac{1}{s^0(s+1)} \quad ; \quad G(s) = \frac{k}{s^2+2s+3}$$

Type #1 System:

eg:

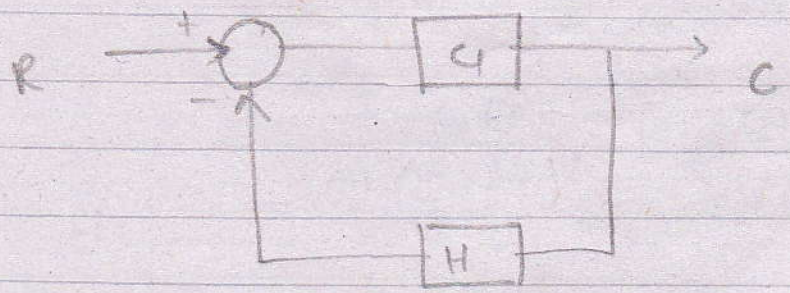
$$G(s) = \frac{s+2}{s^1(s+3)}$$

Type #2 System:

eg:

$$G(s) = \frac{s+2}{s^2(s^2+2s+2)}$$

Any sys can be represented by the following feedback sys.



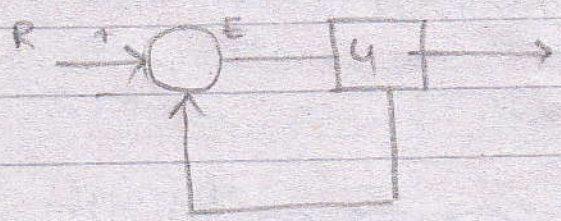
$$\frac{C}{R} = \frac{G}{1+GH}$$

If we want unity feedback, the sys becomes;

$$G_{eq} = \frac{G}{1 - G + GH} \qquad \frac{C}{R} = \frac{G_{eq}}{1 + G_{eq}}$$

Steady-State Error

Consider a unity-gain system



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$\frac{G(s)E(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$E(s) = \frac{R(s)}{1 + G_1(s)} \quad \rightarrow (B)$$

Steady state in time domain

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$= \lim_{s \to 0} sE(s)$$

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)}$$

CASE #1: STEP INPUT

$$r(t) = Ru(t)$$

$$R(s) = R\,\frac{1}{s}$$

$$e_{ss} = \lim_{s \to 0} \frac{sR\ \frac{1}{s}}{1 + G(s)}$$

$$= \frac{R}{1 + \lim_{s \to 0} G(s)}$$

We define

$$K_p = \lim_{s \to 0} G(s) \quad = \text{step error const}$$

$$e_{ss} = \frac{R}{1+K_p}$$

(i) Type Zero System:

from eq (A)

$$K_p = \lim_{s \to 0} \frac{K(1+b_1 s + \cdots)}{s^0(1+a_1 s + \cdots)}$$

$$K_p = K$$

→ We get

$$e_{ss} = \frac{R}{1+K}$$

(ii) Type -1 System

from eq A

$$K_p = \lim_{s \to 0} \frac{K(1+b_1 s + \cdots)}{s^2(1+a_1 s + \cdots)}$$

$$K_p = \infty$$

We get

$$e_{ss} = \frac{R}{1+\infty} = 0$$

* Same
higher

Conclus

For St

→ Type - z
error

→ Type -
error

$e_{ss} =$

* Same for type -2 sys } 3 } other higher order system.

CONCLUSION

FOR STEP INPUT

→ Type-zero has a finite steady state error

→ Type -1, 2, ... have zero " " error.

CASE #2: RAMP INPUT

$$r(t) = Rtu(t)$$

$$R(s) = R\frac{1}{s^2}$$

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot R \cdot 1/s^2}{1+G(s)}$$

$$= \lim_{s\to 0} \frac{R}{s[1+G(s)]}$$

$$= \lim_{s\to 0} \frac{R}{s+sG(s)}$$

$$= \frac{R}{0+\lim_{s\to 0} sG(s)}$$

We define

$$K_v = \lim_{s \to 0} sG(s) = \text{Ramp Error Const.}$$

$$e_{ss} = \frac{R}{K_v}$$

i) Type Zero System

From eq (A)

$$K_v = \lim_{s \to 0} \frac{s \times k(1 + b_1 s + \cdots)}{s^0 (1 + a_1 s + \cdots)}$$

$$K_v = 0$$

then

$$e_{ss} = \frac{R}{0} = \infty$$

ii) Type 1 System

From eq (A)

$$K_v = \lim_{s \to 0} s \times \frac{k(1 + b_1 s + \cdots)}{s(1 + a_1 s + \cdots)}$$

$$K_v = K$$

then

ii) Type 2

from

then

Same fo

CONCLUSION

For ramp

→ Type 0
Ramp i/p
It has
Order sy
Controller

→ Type 1
error

→ Type 2 3

then

$$e_{ss} = \frac{R}{K}$$

iii) Type 2 System

from eqn (A)

$$K_v = \lim_{s \to 0} \frac{s \times K(1+b_1 s + \cdots)}{s^2(1+a_1 s + \cdots)}$$

$$K_v = \infty$$

then

$$e_{ss} = \frac{R}{\infty} = 0$$

Same for higher Order System.

CONCLUSION:

For ramp input:-

→ Type 0 , steady state error is infinite Ramp i/p shud not be given Type 0 sys. It has 2 be converted to a higher Order sys first through some Controller.

→ Type 1 , gives a finite steady-state error for ramp i/p

→ Type 2,3--- have zero steady state error.

CASE #3:

PARABOLIC INPUT

$$r(t) = \frac{Rt^2}{2} u(t)$$

$$R(s) = R^2 \frac{1}{s^3}$$

$$e_{ss} = \lim_{s \to 0} \frac{s \, R \, 1/s^3}{1 + G(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{R}{s^2 + s^2 G(s)}$$

$$= \frac{R}{\lim_{s \to 0} [s^2 + s^2 G(s)]}$$

$$= \frac{R}{0 + \lim_{s \to 0} s^2 G(s)}$$

We define

$$K_a = \lim_{s \to 0} s^2 G(s) = \text{parabolic error constant}$$

$$e_{ss} = \frac{R}{K_a}$$

i) For typ

From eq

$K_a = \lim_{s \to}$

$K_a = 0$

$e_{ss} = \frac{R}{0}$

ii) For type

From eq

$K_a = \lim$

$K_a = 0$

$e_{ss} = \frac{R}{0}$

(iii) For typ

From e

i) For type-zero system

From eq (A)

$$K_a = \lim_{s \to 0} s^2 \times \frac{K(1 + bs + \cdots)}{s^0(1 + a_1 s + \cdots)}$$

$$K_a = 0$$

$$e_{ss} = \frac{R}{0} = \infty$$

ii) For type-1 system

From eq (A)

$$K_a = \lim_{s \to 0} s^2 \times \frac{K(1 + bs + \cdots)}{s^1(1 + a_1 s + \cdots)}$$

$$K_a = 0$$

$$e_{ss} = \frac{R}{0} = \infty$$

(iii) For type-2 sys
from eq (A)

$$K_a = \lim_{s \to 0} \frac{s^2 \times K(1+b_1 s + \cdots)}{s^2(1+a_1 s + \cdots)}$$

$$K_a = K$$

$$e_{ss} = \frac{R}{K}$$

iii) For Type 3 system

From eq (A)

$$K_a = \lim_{s \to 0} \frac{s^2 \times K(1+b_1 s + \cdots)}{s^3(1+a_1 s + \cdots)}$$

$$K_a = \infty$$

$$e_{ss} = \frac{R}{\infty} = 0$$

Summary:

System Type	Error K_p
0	K
1	∞
2	∞
3	∞

Summary:

System Type	Error Const			Steady State Error		
	K_p	K_v	K_a	step input	Ramp input	Parabolic input
0	K	0	0	$\frac{R}{1+K}$	∞	∞
1	∞	K	0	0	R/K	∞
2	∞	∞	K	0	0	R/K
3	∞	∞	∞	0	0	0

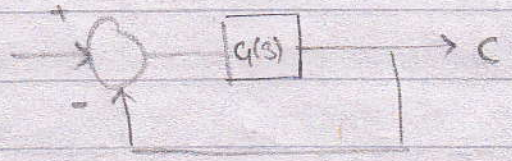
20

Q Determine the step, ramp, and parabolic error const for Unity feed back Control sys.

a) If Unity F.b sys is not given, convert it in2 Unity F.b sys first

in which forward tf is

$$G(s) = \frac{15(s+4)}{s(s+1)(s+2)(s^2+12s+10)}$$



(b)

Using appropriate error Const, obtain [from part (a)] find the steady state error signal for reference input.

$$r(t) = (16+2t)\,u(t)$$

c)

Is the closed loop control sys

a)

$\therefore K_p =$

K_p

K_v

20

a)

$\therefore K_p = \lim_{s \to 0} G(s)$

$= \lim_{s \to 0} \frac{15(s+4)}{s(s+1)(s+2)(s^2+12s+10)}$

$= \frac{1}{0}$

$\boxed{K_p = \infty}$

$K_v = \lim_{s \to 0} sG(s)$

$= \lim_{s \to 0} s \times \frac{15(s+4)}{s(s+1)(s+2)(s^2+12s+10)}$

$= \lim_{s \to 0} \frac{15(s+4)}{(s+1)(s+2)(s^2+12s+10)}$

$= \frac{15(4)}{(1)(2)(10)}$

$= \frac{15 \times 4}{20}$

$\boxed{K_v = 3}$

$$K_a = \lim_{s \to 0} s^2 G(s)$$

$$= \lim_{s \to 0} s^2 \frac{15(s+4)}{s(s+1)(s+2)(s^2+2s+10)}$$

$$k_a = 0$$

b) $r(t) = (16+2t)\,u(t)$

$$e_{ss} = e_{ss_1} + e_{ss_2}$$

$$e_{ss_1} = \frac{R}{1+k_p} = \frac{16}{1+\infty} = 0$$

$$e_{ss_2} = \frac{R}{k_v} = \frac{2}{3}$$

$$e_{ss} = \frac{2}{3}$$

System's characteristic equation is $\frac{C}{R}$'s denominator

$$= 1 + \frac{15(s+4)}{s(s+1)(s+2)(s^2+2s+10)}$$

$$= \frac{s(s+1)(s+2)(s^2+2s+10) + 15s + 60}{s(s+1)(s^3+2s^2+10s+2s^2+4s+20)}$$

$s(s+1)($

s^5+2

s^5+

Apply

s^5

s^4

s^3

s^2

s^1

s^0

$s(s+1)(s^3+2s^2+10s+2s^2+4s+20)=0$

$s^5+4s^4+14s^3+20s^2+s^4+4s^3+14s^2=0$
$+20s+15s+60$

$s^5+5s^4+18s^3+34s^2+35s+60=0$

Apply Routh's algorithm:

s^5	1	18	35
s^4	5	34	60
s^3			
s^2			
s^1			
s^0			

s+10)
+15s+50
+20)

Root Locus

The characteristic eq of a transient response of a closed loop system is related to the location of closed loop poles depends on the value of loop gain to be collected. Hence the design shd behav a knowledge how a closed loop poles moved in s-plane as the loop gain is vary. From the design point of view in some systems simple gain adjustments may move the closed loop poles to the desired locations Hence the design problem will become the selection of an appropriate gain value.

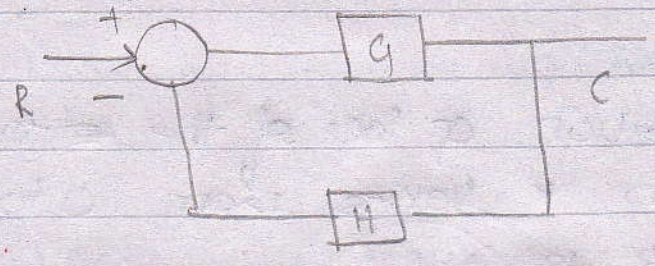
In this regard an imp study in linear control system theory is the investigation of trajectories of the roots of the characteristic eq or "Root locii", when a certain parameter varies. This simple technique known as root locus technique.

In other
the part
eq as
range
transfe

$1+GH=0$

By posi
stabilit

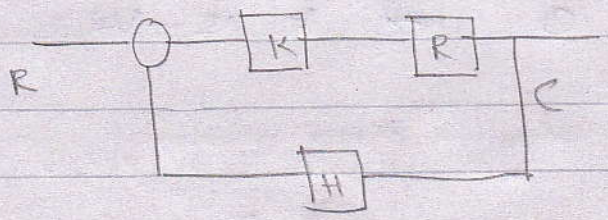
In other words the root locci is the part of roots of characteristic eq as a function of gain 'k' with range $(-\infty < k < \infty)$ of open loop transfer funct



$$\frac{C}{R} = \frac{G}{1+GH}$$

$1+GH = 0 \longrightarrow$ characteristic eq

By portion of zeros we find Sys stability of



$$\frac{C}{R} = \frac{KG}{1+KGH}$$

$$1 + KG + 1 = 0$$

- Unity feedback sys can be represented in non-unit feedback
- Non-Unity " " " " "
  in Unity system

Root Loci

The portion of loci of the ~~roots~~ roots when K varies from 0 to ∞ ie K is +ve

Complementary Root loci

The portion of loci when K varies -∞ to 0 ie K is -ve.

* Complementary Roots Locci

The portion of locci when k varies -∞ to 0 ie k is -ve.

$$G(s) = \frac{K}{s(s+1)(s+3)} \quad \text{or} \quad (s+1)(s+3)s = 0$$

1) $K=0$ pts, $s=0, -1, -1$ $\quad n=3$

$K=\pm\infty$ pts, $s=\infty, \infty, \infty$ $\quad m=0$

2) No of branches = 3 (equal to total no of poles)

3) Entire root locus is symmetrical wrt real axis

4) Centroid:

$$\sigma = \frac{\Sigma(\text{Finite poles}) - \Sigma(\text{Finite zeros})}{|n-m|}$$

$$= \frac{(0-1-2) - (0)}{|3-0|} = -\frac{3}{3}$$

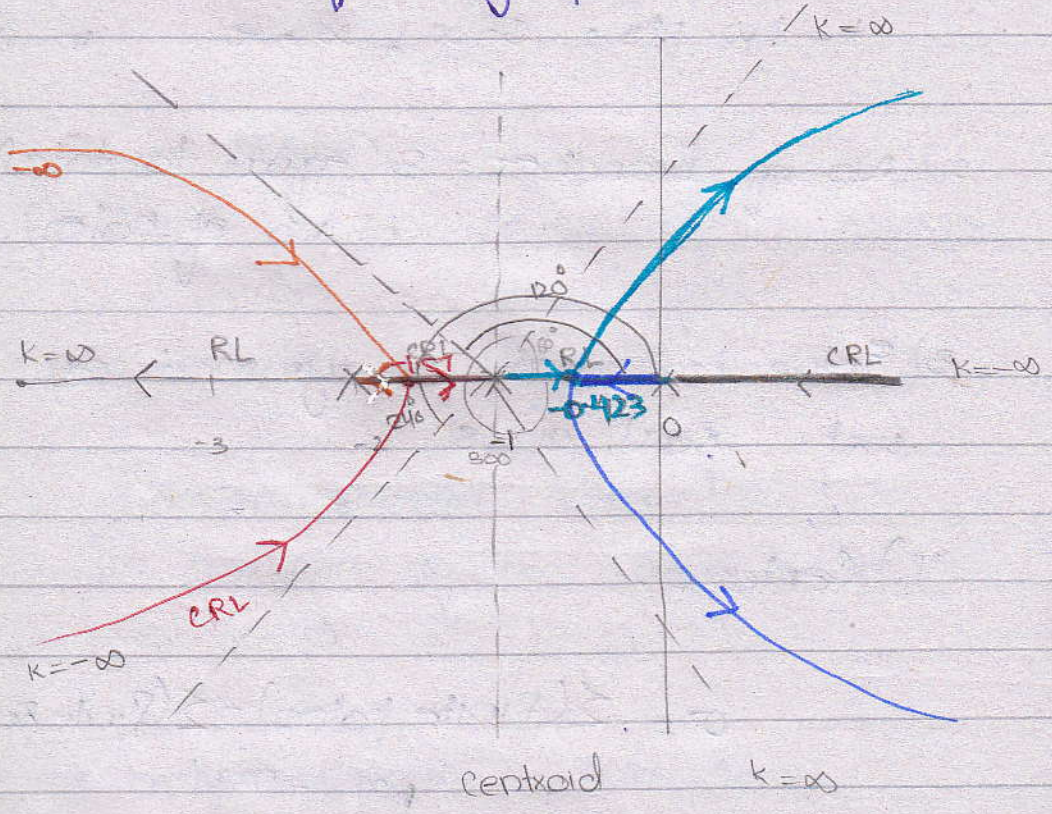
$$\boxed{\sigma = -1}$$

5) No of Asymptotes

For RL: $|n-m| = |3-0| = 3$

for CRL: $|n-m| = |3-0| = 3$

total no of assymptotes = 3+3=6



Angle of Asymptotes

FOR RL:

$$\theta_{RL} = \frac{2h+1}{|n-m|} \times 180^\circ$$

Where

$h = 0, 1, 2, \cdots |n-m| - 1$

$h = 0, 1, 2$

$$\theta_{RL} = \frac{2k+1}{3} \times 180^\circ = 60^\circ, 180^\circ, 300^\circ$$

FOR CRL

$$\theta_{CRL} = \frac{2\pi}{|n-m|} \times 180^\circ$$

$$\boxed{\theta_{CRL} = 0, 120^\circ, 240^\circ}$$

Break away / Break in points

$$1 + G(s) = 0$$

$$1 + \frac{K}{s(s+1)(s+2)} = 0$$

$$s(s^2+3s+2) + K = 0$$

$$-K = s^3 + 3s^2 + 2s$$

$$\frac{d(-K)}{ds} = 3s^2 + 6s + 2 = 0$$

$s = -1.57$ — selects value for CRL

$s = -0.423$ — " " for RL.

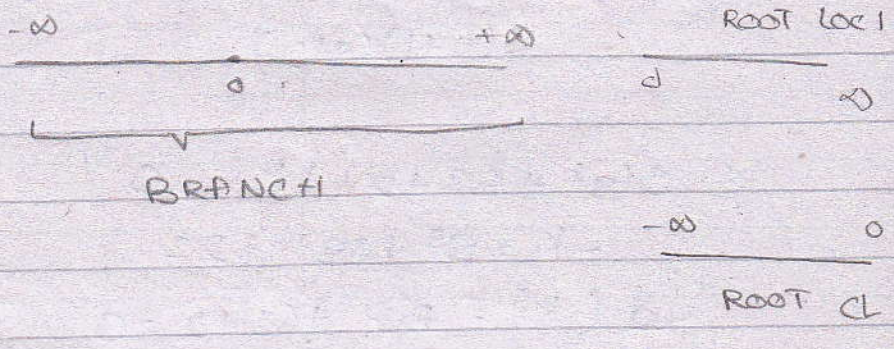
If $3(s+1)(s+2)+k=0$ is given we have to find $G(s)$ then

$$\frac{3(s+1)(s+2)+k}{3(s+1)(s+2)} = 0$$

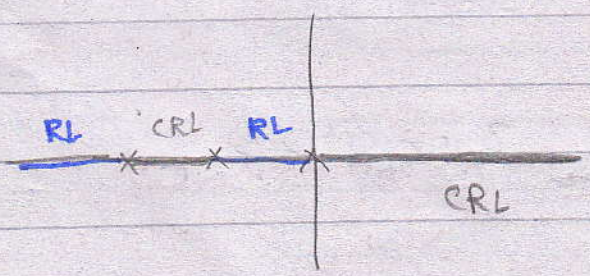
$$1 + \frac{k}{3(s+1)(s+2)} = 0$$

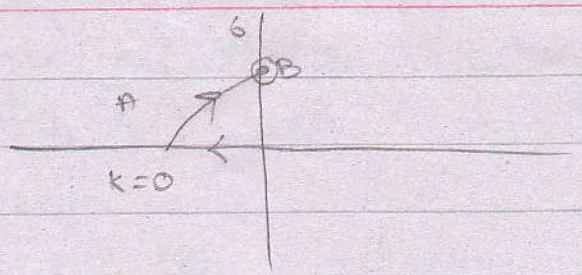
By comparing $1+G(s)=0$

$$G(s) = \frac{k}{3(s+1)(s+2)}$$



* Distribution RL & CRL in real axis
short cut $s=-4$





marginally stable sys here

$K = 6$

Imaginary axis crossing point

$$1 + G(s) = 0 \Rightarrow s(s+1)(s+2) + k = 0$$

$$s^3 + 3s^2 + 2s + k.$$

s^3	1	2
s^2	3	$k \Rightarrow A(s)$
s^1	$\frac{6-k}{3}$	
s^0	k	

For system to be stable the first column must be greater than zero

NOTE FOR PAPER:

* Do form SFG of blk diagram Convert into SFG & find $\frac{C}{R}$ and then match with the $\frac{C}{R}$ obt from blk diag.

So

$$\frac{6-k}{3} > 0$$

$$k < 6$$

$0 < k \leq 6 \rightarrow$ system is stable

$$A(s) = 3s^2 + 6 = 0$$

$$s = \pm j\sqrt{2}$$

Angle of departure & Angle of arrival

Σ(angle of zeros) $-$ Σ(angle of poles

$= 180^\circ$

This

CONCLUSI

- When magnitu oscilla
- The s Underd requir show
- If we Overda then of K the s

a) s(s+

3) G(s)

4) s(

5) s(

This is for complex poles

CONCLUSION:

- When $k = 6$ system will behave as magnitally stable with sustained oscillations with freq $\sqrt{2}$ rad/sec
- The sys will be designed as underdamped sys acc to the sys requirement on the trajectory shown b/w point A & B
- If we want to design a sys overdamped or critically damped then we have to choose the value of k at wch the poles of the sys will be -ve & real value

a) $s(s+2)(s^2+2s+2)+k=0$

3) $G(s)H(s) = \dfrac{K(s+1)}{s(s+4)(s^2+2s+2)}$

4) $s(s^2+2s+2)+k=0$

5) $s(s+3)(s^2+2s+2)+k=0$

* Mode of SFG of every blk diagram & Verify c/R of blk diag by CLR and SFG

* Qs from Chap 2 & 3

2) $s(s+2)(s^2+2s+2)+k=0$

$$\frac{s(s+2)(s^2+2s+2)+K}{s(s+2)(s^2+2s+2)} = 0$$

$$1 + \frac{K}{s(s+2)(s^2+2s+2)} = 0$$

$$\therefore G(s) = \frac{K}{s(s+2)(s^2+2s+2)}$$

$s(s+2)(s^2+2s+2)=0$

$K=0$, $s=0,-2,-1\pm j$ $n=4$

$K=\infty$, $s=\infty,\infty,\infty$ $m=0$

Centroid

$$\sigma = \frac{\sum (0-2-1+j-1-j)}{|4-0|}$$

$$= \frac{-4}{4} = -1$$